FDA - CAD

FLIGHT DYNAMICS ANALYSIS - COMMAND AUGMENTATION DESIGN

Version 3.01

USER GUIDE

A Matlab Graphical User Interface for Flight Dynamics Analysis

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February 2013

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1 GETTING STARTED WITH FDA-CAD

1.1 System Requirements

Matlab® Version 7.0.4 or higher.

Matlab® Control Toolbox Version 6.2 or higher.

Adobe® Acrobat® Reader Version 8 or higher.

1.2 Installation

Copy all the files to a directory of your choice - preferably a new folder named FDA-CAD. Within Matlab, add this folder to the Matlab path. This is achieved by selecting the 'File' Menu and choose ‘Set Path…’ click on 'Add Folder', using the file browser, select the newly created folder and click 'OK'. Alternatively, the folder can be added by selecting it to be the temporary current Matlab directory from the option in the Workspace Window toolbar.

1.3 Execution

Within Matlab, set the 'Current Directory' to the newly created folder.

From the Matlab Command Window type 'fdacad3' and press enter to start the program.

1.4 Screen Size

FDA-CAD was designed to operate on a screen with size properties,

- Screen resolution 1024×768 pixels
- Screen aspect ratio 4:3
- Screen physical size 12×9 inches (305×229mm)

1.3 Directory Path

In order that the **“About”** and **“User Guide”** *.PDF files can be opened from within FDA-CAD, it is necessary to change the path information in the program. Open the “**fdacad3.m**” file in the Matlab Editor and enter the correct path for your installation in lines 586, 590 and 594.

2. FDA-CAD OVERVIEW

2.1 Flight Dynamics Analysis

Version 3.0 of FDA-CAD is designed explicitly to accompany the book *Flight Dynamics Principles*[1] (FDP) by the author. The program facilitates the rapid solution and analysis of the linear equations of motion of an aircraft and incorporates tools that accommodate various notational styles. The notation and symbols correlate with those given in FDP as far as the limitations of the program language permit. In general, interpretation of the GUI screens and the notation adopted should be obvious to those familiar with the subject.

FDA-CAD facilitates Longitudinal and Lateral-Directional stability and control analysis of an aircraft given the flight condition data and stability and control derivatives referenced to either wind or body axes. Results obtained from the analysis include the matrix state equation, matrix output equation, response transfer functions, time history plots, system pole and zero descriptions, and stability mode characteristics. A derivative conversion feature enables the user to calculate derivative values in different formats and referred to alternative reference axes.

Stability augmentation system design tools include provision for the inclusion of an actuator model and root locus, or pole placement, computation for designing feedback gains. Longitudinal analysis of the basic aircraft may be undertaken using either the full order model or the reduced order model, but with this version of FDA-CAD, longitudinal stability augmentation design may only be undertaken with the full order model. All lateral-directional analysis can only be undertaken with the full order model. The stability augmentation analysis tools also permit the direct comparison of unaugmented and augmented time response plots on the same axes for the selected output variables.

Longitudinal closed loop system design permits feedback path to elevator gain design for axial velocity K_u, normal velocity K_w, pitch rate K_q and pitch attitude K_θ in any combination. The design of feedback loop gains to the thrust input is beyond the scope of version 3.0 of

FDA-CAD. Longitudinal handling qualities assessment and command path design are also beyond the scope of version 3.0 of FDA-CAD.

Lateral-Directional closed loop system design and analysis provides feedback path gains for sideslip β, roll rate p, yaw rate r and roll attitude ϕ to aileron and, or rudder. Lateral closed loop feedback to aileron and directional closed loop feedback to rudder are dealt with on two different screens. The directional screen also incorporates the facility for designing the aileron-rudder interlink gain. The lateral-directional loops may be closed in any combination to suit the problem at hand.

2.2 Graphical User Interface (GUI)

Most of the Graphical User Interface (GUI) objects have 'tool-tips' which are revealed whenever the mouse pointer is held over a field, button, pop-up menu, etc. For example, typical tips include the function performed by a button, the current units applicable to the aircraft data, definition of a derivative and other advisory information. Warning dialogue boxes are also revealed whenever the user invokes an operation which may give rise to an erroneous or irreversible outcome.

3. THE OPENING AND BASIC AIRCRAFT SCREENS

3.1 The Opening Screen

The opening screen shows imagery linking the program to the book FDP. Four buttons appear on the screen:

'About FDA-CAD':- opens a short PDF document describing the history and scope of the program.

'Longitudinal Analysis':- transfers the user immediately to the longitudinal basic aircraft analysis screen and resets the variable data fields to zero.

'Lateral-Directional Analysis':- transfers the user immediately to the lateral basic aircraft analysis screen and resets the variable data fields to zero. The user may then toggle between lateral or directional analysis from the lateral screen.

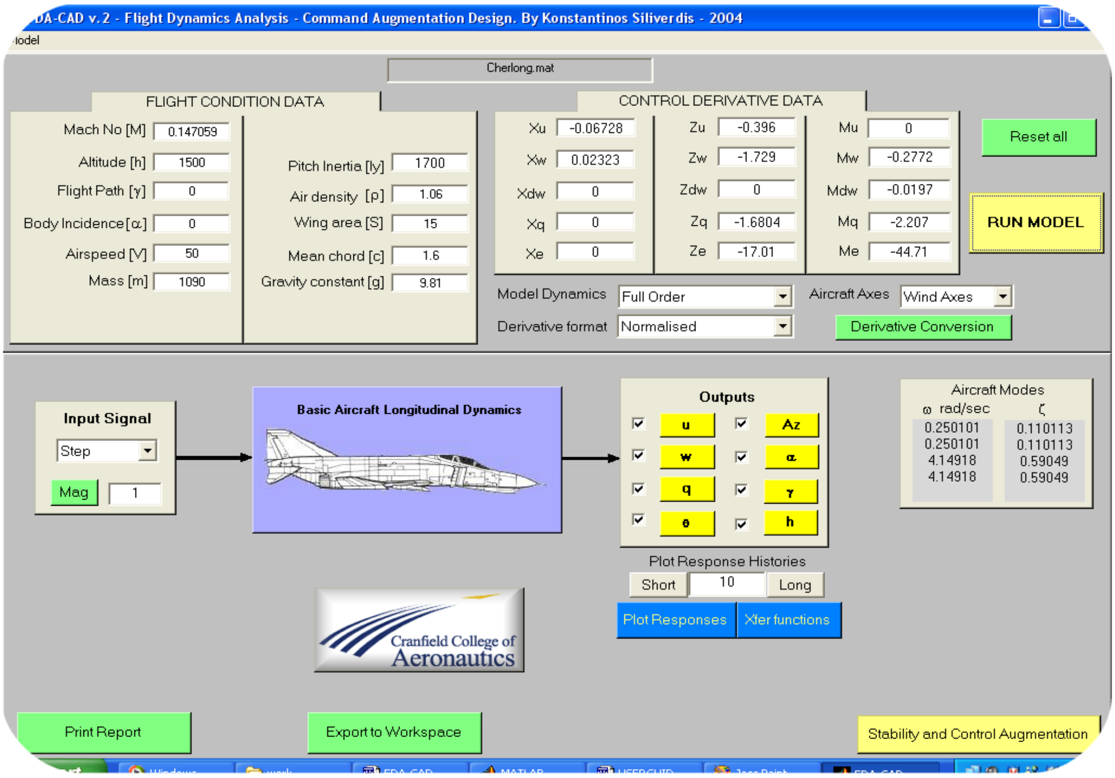
'Open User Guide':- opens the PDF file containing this document.

The top of screen menu bar includes the '**FDA-CAD help'** button which may also be used to open the User Guide. This appears on all of the screens. The program may be shut down from the opening screen by clicking on the top right hand title bar button in the usual way.

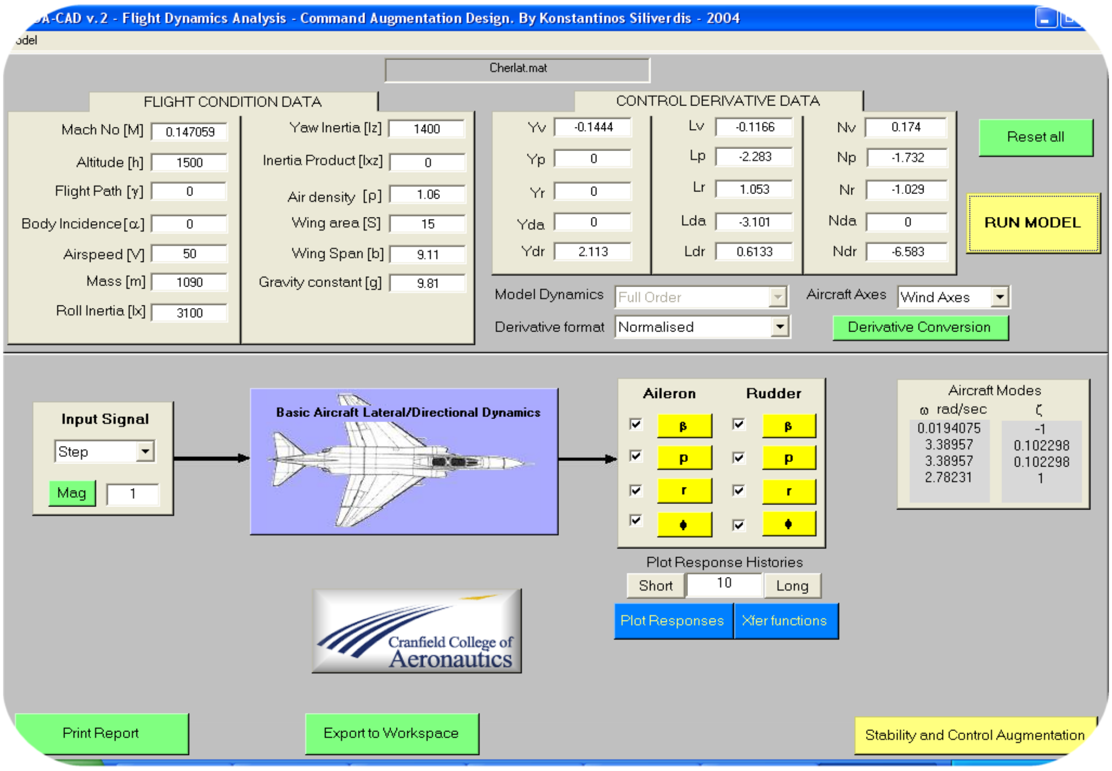
3.2 The Basic Aircraft Screens

The longitudinal and lateral-directional basic aircraft screens are very similar in appearance. The user choice simply sets the annotation of the screen as required. The longitudinal and lateral-directional screens are shown below. In the interests of expediency, these images are those developed for version 2.0 of FDA-CAD and differ in minor detail only from those developed for version 3.0. The most notable difference is the absence of the large College of Aeronautics button which has been replaced by a smaller '**Return to opening screen'** button at the bottom of the window. Other minor changes are mostly cosmetic to improve overall appearance. The functional properties of the screen objects are described in the following paragraphs.

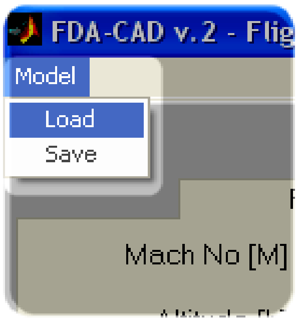
The longitudinal basic aircraft screen presentation



The lateral-directional basic aircraft screen presentation



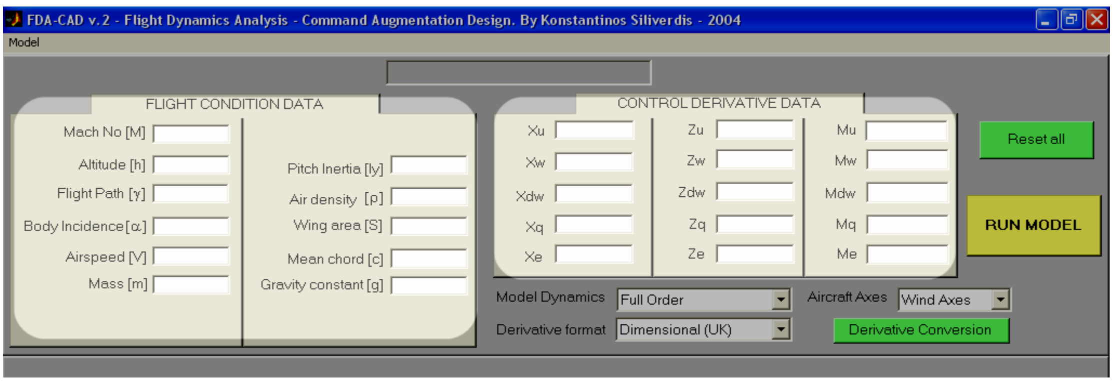
3.2.1 Menu options



The top of screen menu bar shows the '**Model'** button as well as the '**FDA-CAD help'** button. Selecting '**Load'** from the '**Model'** menu opens the standard Windows® browse window from which a ***'*.mat'*** file containing previously saved aircraft data may be selected. Select the desired file and click '**Open'**. This action sets longitudinal or lateral-directional analysis from saved information as appropriate, it loads the aircraft data and places the filename in the title field at the top of the GUI. This data is then visible in the flight condition and derivative data fields in the top half of the screen. Alternatively, data can be entered by hand and the '**Save**' button will bring up the Windows® save window in which the file name and save destination can be entered. The model data can be saved at any time the screen is visible and when a model is saved, its name appears in the title field of all analysis screens as reference.

3.2.2 Data entry

The input fields are situated in the top of the screen and are separated into flight condition and control derivative data. Numerical data may be entered into each field from the keyboard. Once a field value is filled continue to the next using the mouse. An entry must be made in each field. If a field is not completed or its input is not a valid number the



program will return a warning message. Individual data entries can be corrected or changed at any time. It is critically important that the user ensures consistent units for all the data entered into the model. The **'RESET ALL'** button clears the entire model data and sets all fields to a blank. A new model can then be entered manually or loaded from a data file. The data is also cleared by pressing the **'Return to opening screen'** button, and this action is advised before starting work with a new aircraft model.

3.2.3 Derivative format selection

When data is entered manually, the derivative format and aircraft reference axes must be set using the drop down menus. Three alternative derivative formats are available which

covers most applications – '**Dimensional'**, '**Dimensionless (UK)'** and '**Normalised (USA)'** format. When the model data is saved this information is saved with the data. Note that data given in normalised North American format, as found in Teper[2] and in Heffley and Jewell[3] may be entered directly when '**Normalised (USA)'** format is selected.

3.2.4 Model order selection

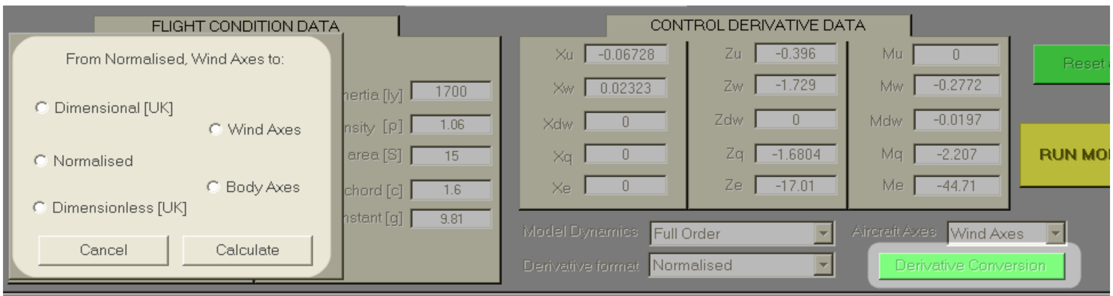
From the Model Dynamics pop-up menu the user can choose between '**Full Order'** or '**Reduced Order'** analysis for the longitudinal aircraft model only. The **'Full Order Only'** model is used for lateral-directional analysis when the pop-up menu window is deactivated.

3.2.5 Aircraft axes selection

The pop-up axes selection menu choice does not influence the model analysis directly, but it is used in the derivative conversion process. For the model analysis process the axes notation is made clear by the presence or absence of a non-zero body incidence value.

3.2.6 Stability and control derivative conversion

By pressing the '**Derivative Conversion**' button, a new menu appears to the left of the screen where the user may choose a particular derivative conversion. By pressing the '**Calculate**' button a report is generated in the Matlab command window, an example of which is shown below.



==

DERIVATIVE CONVERSION REPORT FOR: Cherlong.mat

==

General Flight Conditions:
Mach: 0.15
Height: 1500 ft
Body Incidence: 0.00 deg

Original Axes notation: WIND AXES

=======Normalised [UK] Derivatives in Wind Axes=======

Xu=	-0.06728	Zu=	-0.39600	Mu=	0.00000
Xw=	0.02323	Zw=	-1.72900	Mw=	-0.27720
Xdw=	0.00000	Zdw=	0.00000	Mdw=	-0.01970
Xq=	0.00000	Zq=	-1.68040	Mq=	-2.20700
Xeta=	0.00000	Zeta=	-17.01000	Meta=	-44.71000

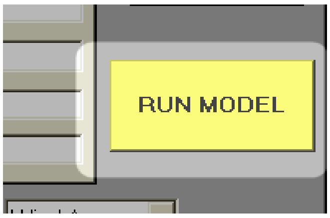
=======Dimensionless [UK] Derivatives in Wind Axes=======

Xu=	-0.18449	Zu=	-1.08589	Mu=	0.00000
Xw=	0.06370	Zw=	-4.74116	Mw=	-0.74094
Xdw=	0.00000	Zdw=	0.00000	Mdw=	-1.64554
Xq=	0.00000	Zq=	-2.87993	Mq=	-3.68701
Xeta=	0.00000	Zeta=	-0.93288	Meta=	-2.39016

The conversion does not change the visible model used in the on screen analysis. Its purpose is to provide a convenient “off-line” tool for use when derivative values in an alternative format are required by the investigator. It is essential to exercise care when using this tool as some conversions will require a non-zero value for the body incidence (α) data field, which will of course be set to zero if the original model data is referred to wind axes. It is prudent to return the incidence value to its initial model value after the conversion calculation is completed to avoid errors in any further interactive analysis. A warning dialogue window opens in these situations to provide a reminder.

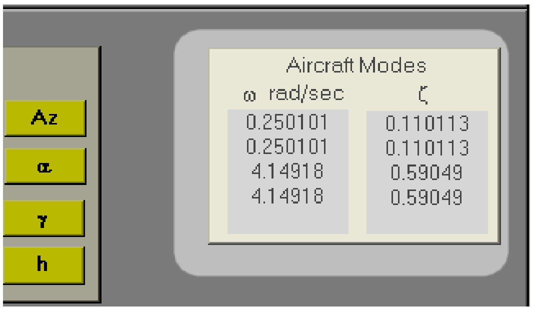
3.2.7 Solution of the equations of motion

By pressing the '**RUN MODEL**' button the aircraft equations of motion are solved for the given input data. If a wind axes notation is selected and the body-incidence is non-zero, a warning message is displayed before proceeding. This function can be invoked as many times as necessary when, for example, a flight data or derivative value is corrected or changed..



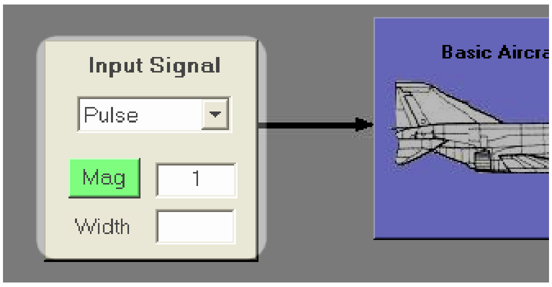
3.2.8 Stability modes

When the equations of motion are solved, the stability modes characteristics are shown in the aircraft modes window. The mode frequencies and damping ratios are shown. When the longitudinal reduced order solution is selected then the short period mode characteristics are shown together with the pitch attitude lag, since the latter variable is retained in the model to ensure consistency with both wind and body axes referenced solutions.

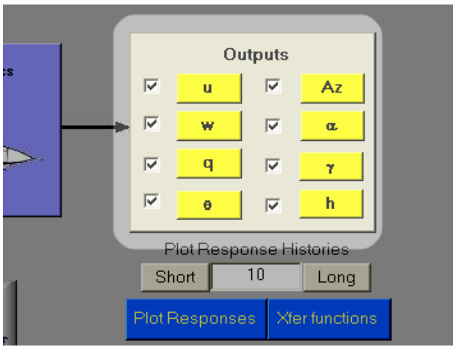


3.2.9 Input signal options

This pop-up menu provides the user with a choice of input signal types. These include; '**Step'**, '**Impulse'**, '**Pulse'**, '**Ramp'**, '**Sine wave'** and '**Doublet'**. The input signal is taken to mean aileron, elevator or rudder angle, depending on context, and the default units are radians. The input signal properties can be can be selected, which includes the magnitude in radians, the pulse or doublet width in seconds and the sine wave frequency in radians per second. The step input is the default input signal and the '**Mag**' button toggles between an input magnitude of 1 radian or 1 degree, expressed in radians (0.0175). The user can also enter an alternative value in the magnitude field.



3.2.10 Response options



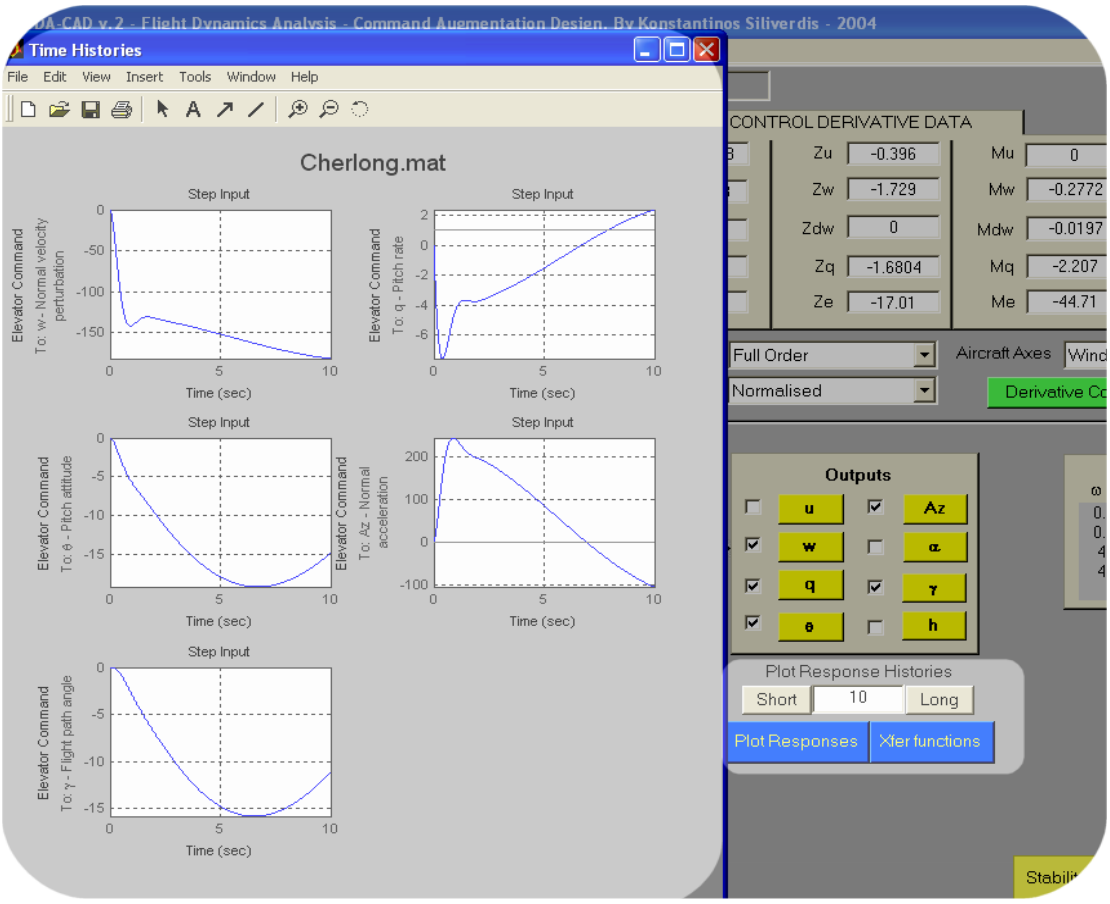
The **'Outputs'** window shows the available variables from which the response time history plots may be selected. Only the checked (✓) responses will appear in the response plots and these may be toggled on or off. By pressing the yellow buttons a Matlab LTI figure window opens to show the Bode plot for the selected variable. The Bode plot can then be printed or saved from this window in the usual way. Whilst the LTI figure window is open, the user may also obtain alternative plots such as, Nichols, Nyquist, Step, Singular value, etc, which can also be saved and printed.

Important: Some aircraft transfer functions are negative and some are non-minimum phase. To avoid problems with frequency response interpretation, transfer function sign is checked and the sign of negative transfer functions is ignored. Thus all frequency response plots are made with positive transfer functions. It is strongly recommended that the user refers to the "correct" transfer function properties, obtained by pressing the **'Print Report'** button, when interpreting Bode and other frequency response plots. See chapters 6 and 7 of FDP[1] for more information on this topic.

3.2.11 Plotting options

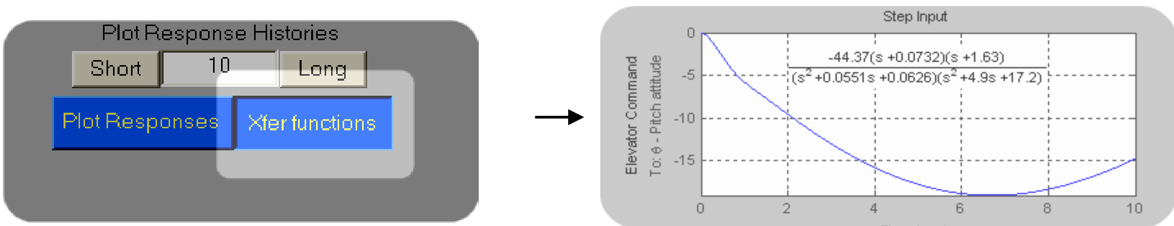
By selecting the response time below the **'Outputs'** window and pressing the '**Plot Responses**' button, a new Matlab figure window appears containing the checked response time histories. The figure title is shown as the file name of the aircraft model. A typical example is shown below. The **'Short Period'** and **'Long Period'** buttons select a response time of 10 seconds or 100 seconds respectively. However, the user may also enter an alternative response time in the field window. A maximum of eight plots may be shown in the figure window. By selecting a smaller number of response plots and the most suitable **'Input Signal'** the size of each plot is increased to fit the figure window thereby enhancing the analytical usefulness. Thus for good resolution check only those variables of primary interest. The figure window may then be saved and printed as required.

The typical response plot figure window



3.2.12 Transfer function inclusion

By toggling the '**Show Transfer Functions**' button before pressing the **'Plot Responses'** button, the transfer functions of each of the response plots is shown in the plot. This is illustrated below.



3.2.13 FDA-CAD report

The '**Print Report**' button generates a report in the Matlab command window from where it can be printed and saved as required. The report shows the equations of motion of the

basic aircraft in state space matrix format, the factorised transfer functions obtained in the solution of the equations of motion and the stability mode characteristics. An example report is shown below.

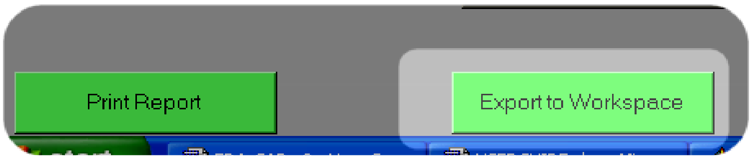
Note: in the example shown *height* h should read *height rate* dh. This is corrected in version 3.0 of FDA-CAD.

```
*--------------------*
*  xdot = Ax + Bu    *
*     y = Cx + Du    *
*--------------------*
u^T = [      Eta          ]
x^T = [      u              w              q         theta ]
Control Law :-
eta_d = delta - K [u]

K =

     0      0      0      0

STATE SPACE MATRICES :-

A =

     -0.06728       0.02323             0         -9.81
       -0.396        -1.729         48.32             0
    0.0078012      -0.24314       -3.1589             0
            0             0             1             0


B_transposed =

            0        -17.01       -44.375             0


C =

            1             0             0             0
            0             1             0             0
            0             0             1             0
            0             0             0             1
       -0.396        -1.729       -1.6804             0
            0          0.02             0             0
```

Continued below…

```
          0        -0.02            0           1
          0           -1            0          50


D_transposed =

     0     0     0     0     0     0     0     0


TRANSFER FUNCTIONS - FACTORISED FORM

Zero/pole/gain from input to output "u - Axial velocity perturbation":
          -0.39514 (s-974.3) (s+1.85)
-------------------------------------------------
(s^2  + 0.05508s + 0.06255) (s^2  + 4.9s + 17.22)


Zero/pole/gain from input to output "w - Normal velocity perturbation":
  -17.01 (s+129.2) (s^2  + 0.06667s + 0.07902)
-------------------------------------------------
(s^2  + 0.05508s + 0.06255) (s^2  + 4.9s + 17.22)


Zero/pole/gain from input to output "q - Pitch rate":
        -44.3749 s (s+1.63) (s+0.07321)
-------------------------------------------------
(s^2  + 0.05508s + 0.06255) (s^2  + 4.9s + 17.22)


Zero/pole/gain from input to output "theta - Pitch attitude":
         -44.3749 (s+1.63) (s+0.07321)
-------------------------------------------------
(s^2  + 0.05508s + 0.06255) (s^2  + 4.9s + 17.22)


Zero/pole/gain from input to output "Az - Normal acceleration":
 103.9779 (s+37.76) (s^2  + 0.02851s + 0.004665)
-------------------------------------------------
(s^2  + 0.05508s + 0.06255) (s^2  + 4.9s + 17.22)


Zero/pole/gain from input to output "alpha - Angle of attack":
  -0.3402 (s+129.2) (s^2  + 0.06667s + 0.07902)
-------------------------------------------------
(s^2  + 0.05508s + 0.06255) (s^2  + 4.9s + 17.22)


Zero/pole/gain from input to output "gamma - Flight path angle":
     0.3402 (s-15.21) (s+14.03) (s+0.02508)
-------------------------------------------------
(s^2  + 0.05508s + 0.06255) (s^2  + 4.9s + 17.22)


Zero/pole/gain from input to output "h - height":
      17.01 (s-15.21) (s+14.03) (s+0.02508)
-------------------------------------------------
(s^2  + 0.05508s + 0.06255) (s^2  + 4.9s + 17.22)


CHARACTERISTIC POLE LOCATIONS

       Eigenvalue            Damping     Freq. (rad/s)

 -2.75e-002 + 2.49e-001i     1.10e-001      2.50e-001
 -2.75e-002 - 2.49e-001i     1.10e-001      2.50e-001
 -2.45e+000 + 3.35e+000i     5.90e-001      4.15e+000
 -2.45e+000 - 3.35e+000i     5.90e-001      4.15e+000
```

3.2.14 Exporting variables to Matlab workspace



Pressing the **'Export to Workspace'** button places a copy of all the system variables in the Matlab workspace for continuing off-line analysis. The information transferred includes flight condition data, control and stability derivatives, axes notation, derivative format and the transfer function matrix numerators and denominator.

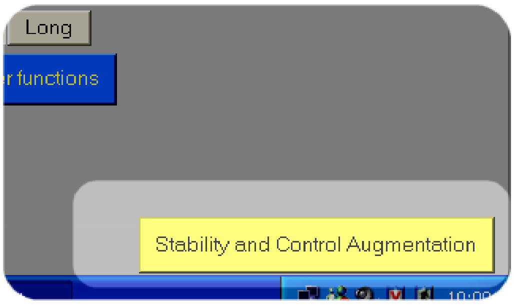
3.2.15 Return to opening screen

Pressing the **'Return to Opening Screen'** button closes down the current aircraft model, resets the data fields to zero and opens the opening screen from which a new study can be initiated.

3.2.16 Close FDA-CAD

Pressing the red **'Close FDA-CAD'** button closes down the entire program. A dialogue box opens first to enable the user to confirm the action. The program may also be shut down by clicking on the top right hand title bar button in the usual way.

3.2.17 Transfer to stability augmentation screen

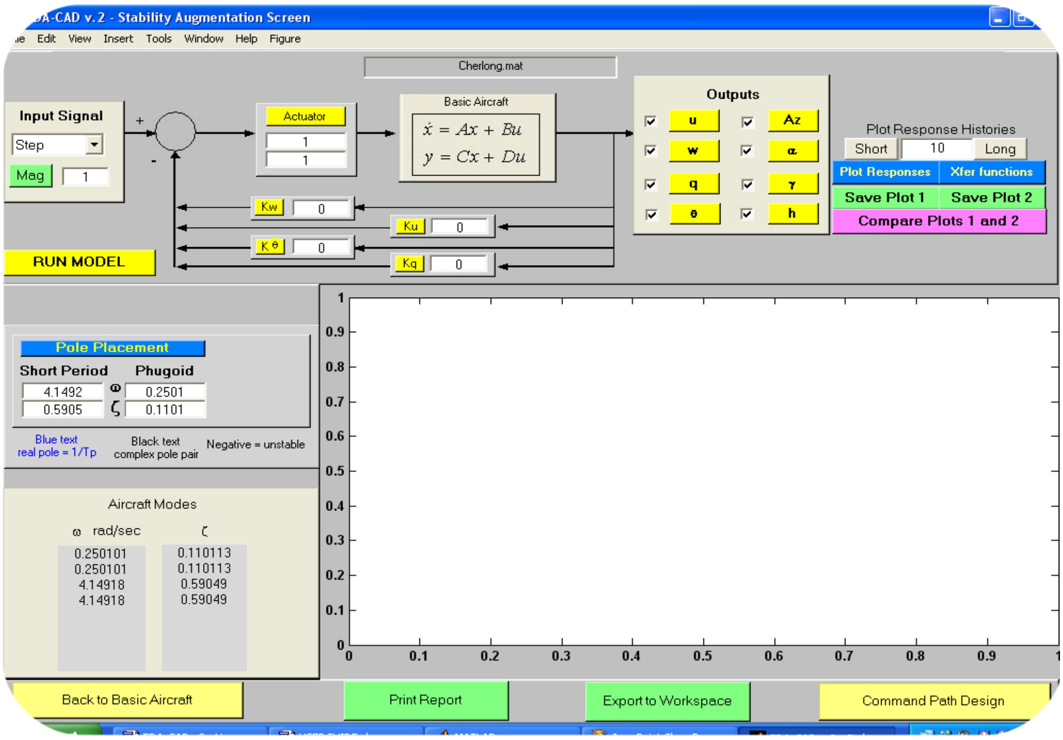


Pressing the '**Stability and Control Augmentation**' button opens a new screen set out for closed loop system design. Before proceeding the program opens a dialogue window, asking whether the user wants to save the current model, or cancel the transfer process and continue working with the basic aircraft. This transfer is not currently available in version 3.0 of FDA-CAD when the user has been working with the longitudinal reduced order aircraft model.

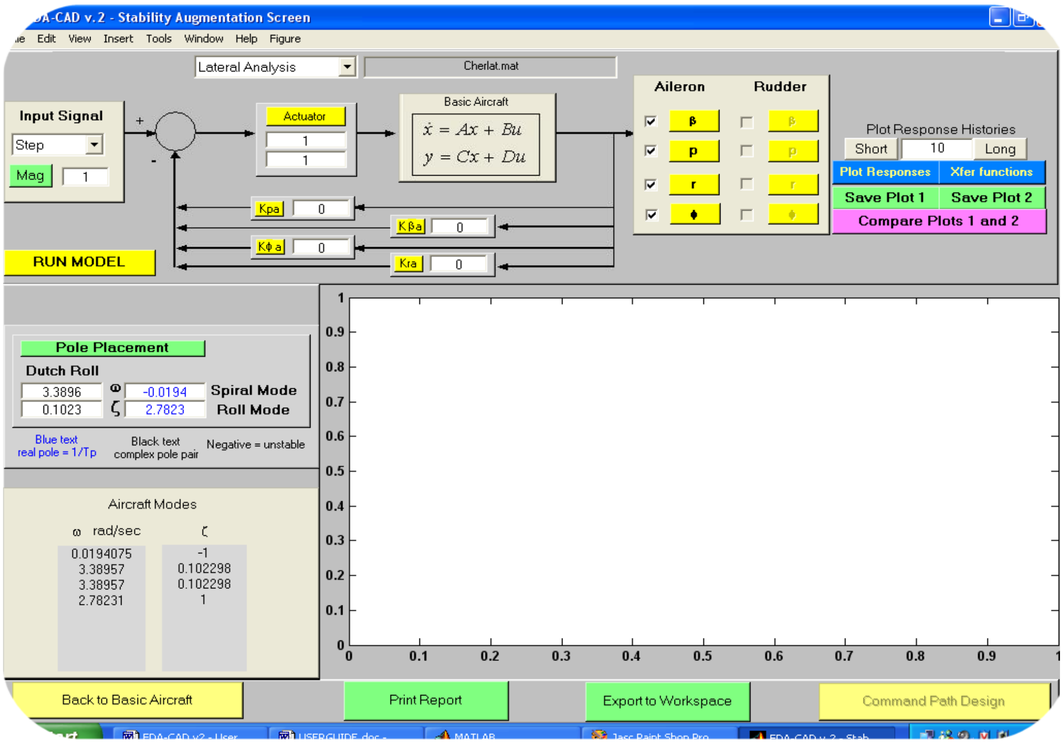
4 THE STABILITY AUGMENTATION SCREEN

In the stability augmentation screen the user may perform simple closed loop feedback gain design, add an actuator to the model and plot and compare time responses for two different system designs. The feedback gains can be designed either by the pole placement technique or by plotting root loci in the reserved plotting area. To avoid excessive congestion in the screen window, for lateral-directional system design the feedback structure is shown separately for the lateral and for the directional applications by selecting the appropriate axis reference from the pop-up menu in the top of the GUI screen to the left of the file name field. The screen appearance is very similar to the basic aircraft screen and many of the functional items are the same. The longitudinal and lateral stability augmentation screens are shown below.

The longitudinal stability augmentation design and analysis screen.

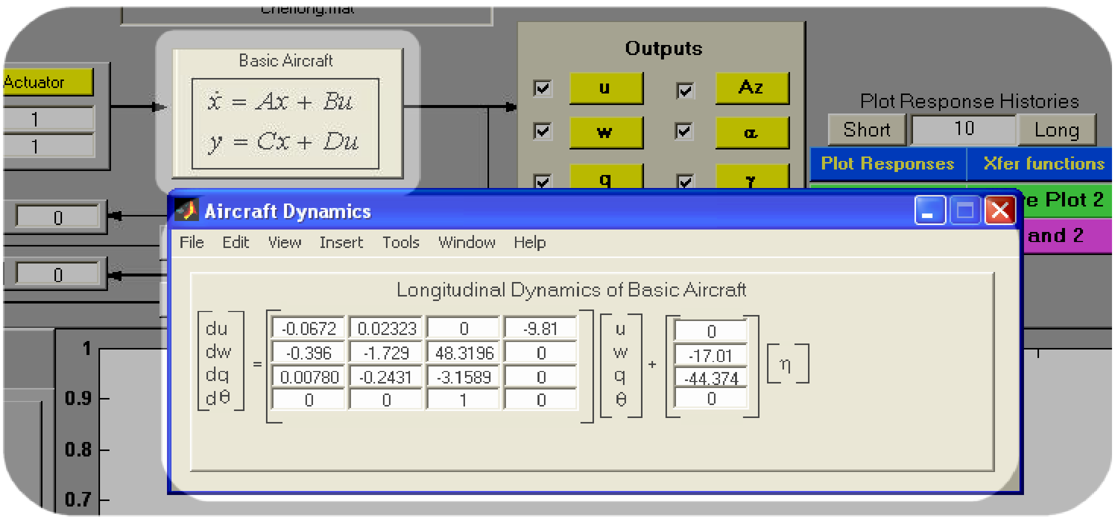


The lateral stability augmentation design and analysis screen.



The version 2.0 screens shown here have been edited to create version 3.0 of FDA-CAD and minor differences will be seen in both presentation and functionality. In particular, version 3.0 does not include any handling qualities analysis tools and the **'Command Path Design'** button and supporting software has been removed.

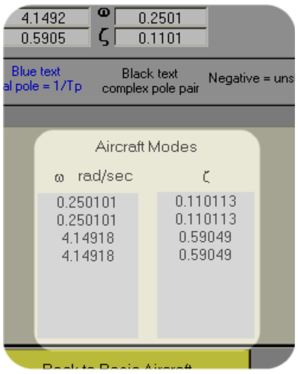
4.1 Basic airframe state space matrices



By pressing the '**Basic Aircraft**' button in the control structure block diagram, the small **Aircraft Dynamics** window opens showing the basic aircraft state equation giving the user direct access to the basic aircraft **A** and **B** matrices. The window is configured such that it can be minimised for repeat opening during the course of a design study.

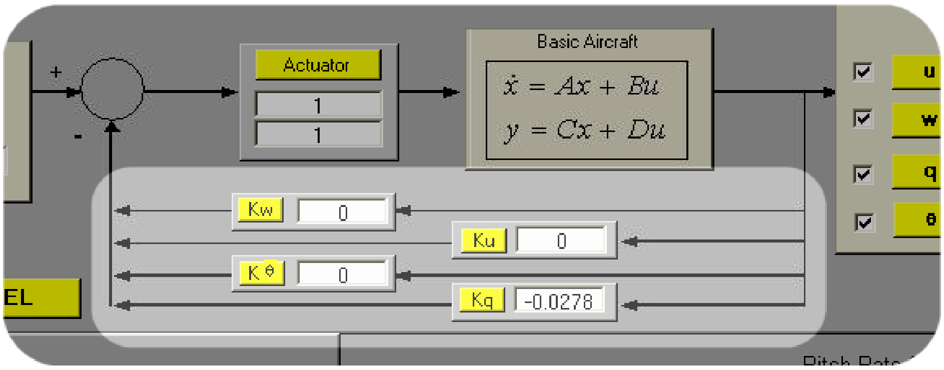
4.2 Stability modes

The stability modes window is located at the lower left part of the GUI screen and its function is the same as in the basic aircraft screen. Every time the '**RUN MODEL**' button is pressed, these fields are updated to show the closed loop modes characteristics, including the actuator poles when included in the model.



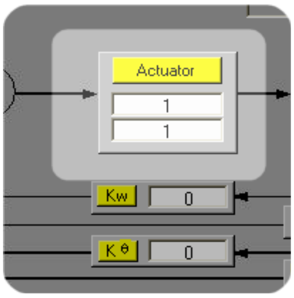
4.3 Feedback gains structure

The feedback structure is configured such that combinations of feedback variables can be selected up to, and including full state feedback. The feedback gains are set at zero until changed by the user. For longitudinal system design the feedback variables are axial



velocity u, normal velocity w, pitch rate q and pitch attitude θ., and these are fed back to elevator through an actuator when selected. For lateral-directional system design the feedback variables are sideslip angle β, roll rate p, yaw rate r and roll attitude ϕ, and these may be fed back to aileron and, or rudder through an actuator when selected. Each yellow feedback path button is labelled with the gain that it represents and by pressing it, its SISO root locus plot appears in the plotting area. Once the required feedback gain variables are determined by the user these are entered manually into the gain fields in the feedback paths. Pressing the **'RUN MODEL'** button calculates the closed loop model which may then be analysed for performance. By this means, feedback loops can be closed and evaluated one-by-one until the desired closed loop system performance is achieved.

4.4 Actuator model

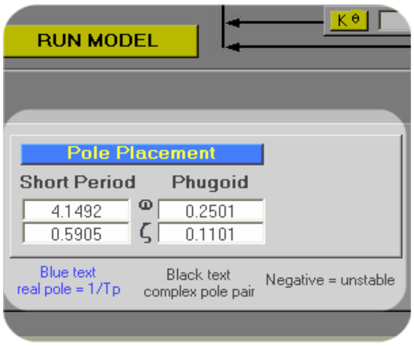


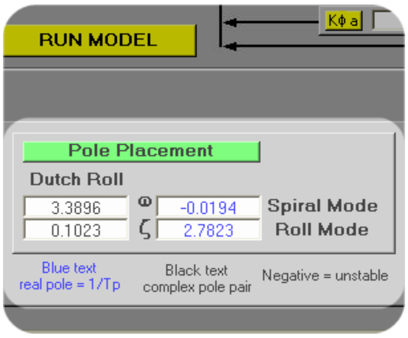
For the actuator model two fields are provided. In the upper field the numerator gain is entered and in the lower field the denominator polynomial is inserted in Matlab format. For example if the actuator transfer function is,

$$F_a(s) = \frac{450}{s^2 + 30s + 450}$$

then the denominator must be inserted as: [1{space}30{space}450], written [1 30 450]. Version 3.0 of FDA-CAD supports a zero order numerator gain and first or second order actuator denominator only. An error message appears on screen if these limits are violated. Pressing the yellow **'Actuator'** button opens the actuator Bode plot in a LTI figure window.

4.5 Pole placement





The general mode characteristics that appear in the aircraft modes fields also appear in the pole placement fields. For the longitudinal case the Short Period and the Phugoid mode characteristics are given, while for the lateral-directional case the Dutch Roll, Spiral and Roll mode characteristics are shown. As the legend states, blue text is used to denote real poles, while a negative sign in either the damping ratio field or in a blue field, denotes an unstable mode. The fields are editable and the user can enter the desired stability mode characteristics for the closed loop aircraft. Then by pressing the '**Pole Placement**' button the feedback gain fields are updated to show the values necessary to achieve the desired dynamics. Pressing the **'RUN MODEL'** button then calculates the solution of the augmented equations of motion and places the closed loop modes characteristics in the relevant fields.

Note: The pole placement tool can not be used when actuator dynamics are included in the model.

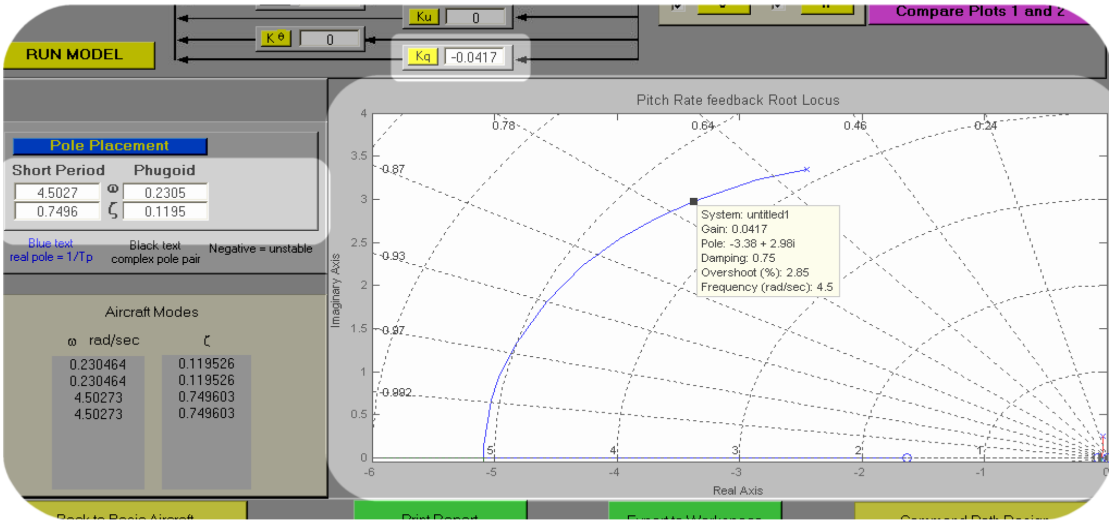
4.6 Feedback gain design using the root locus plot

An alternative method of designing a feedback gain value is by means of a SISO root locus plot. Pressing the yellow button in any feedback path (in the illustration below, K_q) the relevant root locus plot is opened in the reserved plotting window on the screen. The user may then interact with the plot using the standard Matlab root locus plotting tools. By clicking and dragging the mouse on a branch of the locus a text box appears showing the corresponding feedback gain, pole, damping ratio, frequency and overshoot corresponding with that specific locus test point.

Important: Some aircraft transfer functions are negative and give rise to an incorrect root locus plot. To avoid problems with interpretation and correct gain design when using the root locus plot, transfer function sign is checked and the sign of negative transfer functions is ignored. Thus all root locus plots are made with positive transfer functions. When the natural aircraft transfer function is negative, the chosen feedback gain must also be negative to ensure “stabilising” negative feedback. A warning is shown on the root locus screen advising the correct sign of the feedback gain.

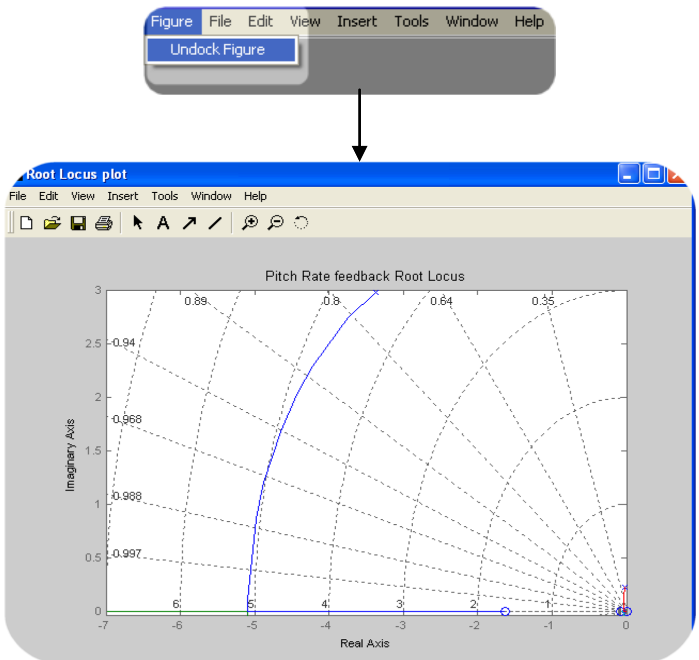
When the pole location corresponding with the desired stability characteristics is chosen the feedback gain value is read from the text box and inserted manually into the appropriate feedback gain field, not forgetting the correct sign. In the illustration below, the aim is to achieve 0.75 damping ratio for the Short Period mode, using pitch rate feedback to elevator. The required feedback gain is shown in the text box as 0.0417. This value is then inserted in the K_q feedback gain field with a ***NEGATIVE*** sign, since the natural aircraft transfer function describing pitch rate response to elevator is negative. Pressing the ‘**RUN MODEL**’ button the aircraft modes fields as well as the Short Period and Phugoid fields are updated to the new closed loop values.

Some limited further adjustment to the root locus plot can be made to improve the analytical accuracy of its interpretation. From a “right click” on the plot, the properties editor can be opened from which some changes can be made.



4.7 Root locus plot undocking

When the user wishes to adjust and annotate the root locus plot further for the purpose of saving or printing, for example, it is convenient to open the plot in a standard Matlab figure window. From the screen menu tool bar click on **‘Root Locus Plot’** and undock the figure. **(‘Figure’** in FDA-CAD version 2.0 as illustrated.) This action opens a new Matlab figure window from which the plot can be saved, printed or further edited.



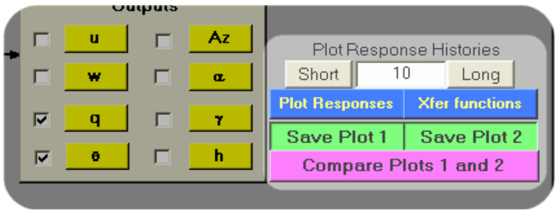
4.8 Input signal options

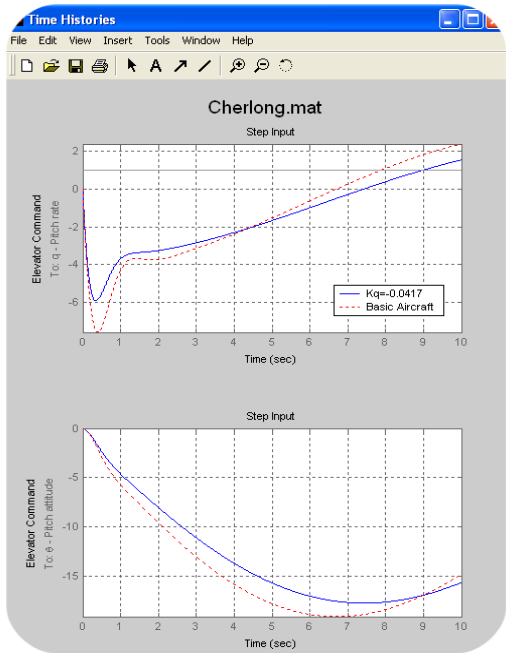
See paragraph **3.2.9 Input signal options** for details.

4.9 Response options

See section **3.2.10 Response options** for details

4.10 Plotting options



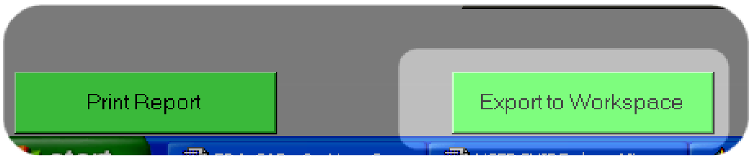


The plotting options are the same as described in paragraph **3.2.11 Plotting options** for the basic aircraft with an additional option for comparing two sets of plots. For this purpose three additional buttons are provided, '**Save Plot 1**', '**Save Plot 2**' and '**Compare Plots 1 and 2**'. By toggling the '**Save Plot 1**' button after making a set of response plots, the figure window is closed and the plot data is stored. If then, a second set of response plots is made for a modified closed loop system model, for example, these may also be stored by toggling the '**Save Plot 2**' button. By pressing the '**Compare Plots 1 and 2**' button a new figure window opens where the checked responses are compared for the two instances as illustrated here. A legend appears on the plots and this may be edited prior to printing if required. Note that when comparing plots the **'Show TF's'** button is deactivated.

4.11 FDA-CAD report

By pressing the '**Print Report**' button, a report similar to that generated from the basic aircraft screen (see paragraph **3.2.13 FDA-CAD report**) is printed in the Matlab command window. The same information is output for the augmented aircraft model and its corresponding feedback gain matrix.

4.12 Exporting variables to Matlab workspace



Pressing the **'Export to Workspace'** button places a copy of all the augmented aircraft system variables in the Matlab workspace for continuing off-line analysis. The information transferred includes flight condition data, control and stability derivatives, axes notation, derivative format, feedback gains and the transfer function matrix numerators and denominator.

4.13 Return to basic aircraft screen

Pressing the **'Return to Basic Aircraft'** button closes down the current augmented aircraft model and returns to the basic aircraft screen and the basic aircraft model.

4.14 Close FDA-CAD

Pressing the red **'Close FDA-CAD'** button closes down the entire program. A dialogue box opens first to enable the user to confirm the action. The program may also be shut down by clicking on the top right hand title bar button in the usual way.

5. EXAMPLES

To illustrate the use of FDA-CAD for problem solving, a number of the worked examples in FDP[1] have been selected to introduce the interactive procedures, capabilities and limitations. Aircraft model data for each example can be found in the *FDP Examples* directory of FDA-CAD v3.01. In each example, the flight condition and derivative data were entered from the keyboard, the axes reference and derivative format were selected, the entries were checked for accuracy and then saved as a model data file (*.mat). Additional data required to complete all the flight data fields were obtained from the reference sources given.

Example 4.2

FDA-CAD is used to show the process of derivative conversion. The flight condition data and dimensionless derivatives should be loaded from the relevant model data example file. The derivative format and axes reference will be correctly set and since the body incidence value is non-zero, this confirms a body axes reference. The units applying are easily identified since the gravity constant g is set to 9.81 m/s^2 , which confirms that SI units apply. Pause the cursor over the data fields to check applicable units.

(i) Press **'Derivative Conversion'** button.
(ii) Derivative conversion menu window opens – select conversion to Dimensional, Body Axes. Press **'Calculate'** button.
(iii) Open Matlab Command Window to see the conversion report, which may be printed or saved as required. See below.
(iv) Compare the dimensional derivatives calculated with those shown in the first set of three equations in Example 4.2.

```
=======================================================
DERIVATIVE CONVERSION REPORT FOR: FDP Example 4-2.mat
=======================================================

General Flight Conditions:
Mach: 0.60
Height: 35000 ft
Body Incidence: 9.40 deg
```

Original Axes notation: BODY AXES

======= Dimensionless [UK] Derivatives in Body Axes =======

Xu=	0.00760	Zu=	-0.72730	Mu=	0.03400
Xw=	0.04830	Zw=	-3.12450	Mw=	-0.21690
Xdw=	0.00000	Zdw=	-0.39970	Mdw=	0.59100
Xq=	0.00000	Zq=	-1.21090	Mq=	-1.27320
Xeta=	0.06180	Zeta=	-0.37410	Meta=	-0.55810

======= Dimensional Derivatives in Body Axes =======

Xu=	12.68597	Zu=	-1214.01427	Mu=	277.46561
Xw=	80.62270	Zw=	-5215.43735	Mw=	-1770.06736
Xdw=	0.00000	Zdw=	-18.32502	Mdw=	-132.47007
Xq=	0.00000	Zq=	-9881.85598	Mq=	-50798.03442
Xeta=	18361.94495	Zeta=	-111152.16188	Meta=	-810703.90627

Example 4.3

FDA-CAD is used to show the process of deriving the longitudinal state equation for an aircraft. With the same aircraft data model loaded as for example 4.2,

(i) Press the **'RUN MODEL'** button. This solves the equations of motion and in the process the longitudinal state equation is calculated.

(ii) Press the **'Print Report'** button. Open the Matlab Command Window to see the solution report. The state equation comprises the state matrix A and the input matrix B, which appear in the first part of the report as shown below.

(iii) Compare the A and B matrices with those in the equation given at the end of Example 4.3.

```
LONGITUDINAL REPORT FOR - FDP Example 4-3.mat

*-------------------*
*  xdot = Ax + Bu   *                    (State Equation)
*     y = Cx + Du   *                    (Output Equation)
*-------------------*
u^T = [     Eta       ]                  (Input variable – elevator angle)
x^T = [     u         w         q        theta ]  (State variables)
Control Law :-
eta_d = delta - K [u]                    (Control law for augmented case equations)

K =                                      (Feedback gain matrix for augmented case equations)

     0     0     0     0
```

STATE SPACE MATRICES :-

A = (State Matrix)

0.00071908	0.0045699	-29.072	-9.6783
-0.068742	-0.29532	174.87	-1.6006
0.0017298	-0.010448	-0.44645	0.0012798
0	0	1	0

B_transposed = (Input Matrix)

1.0408	-6.2939	-4.8885	0

---------- Full solution continues ----------

Example 4.5

FDA-CAD is used to show the process of deriving the longitudinal state equation for an aircraft starting from the American normalised derivative format. Aircraft data as given was entered from the keyboard and flight data not given in the example was obtained from Heffley and Jewell[3]. The model data was saved and this file should be loaded first.

(i) Press the **'RUN MODEL'** button. This solves the equations of motion and in the process the longitudinal state equation is calculated.

(ii) Press the **'Print Report'** button. Open the Matlab Command Window to see the solution report. The state equation comprises the state matrix A and the input matrix B, which appear in the first part of the report as shown below.

(iii) Compare the A and B matrices with those in equation (4.87) in Example 4.5.

(iv) Note that FDA-CAD can only cope with a single input in the longitudinal model - elevator, or stabiliser angle in this example. The second input shown in equation 4.87 is thrust. The thrust response solution could be obtained by replacing the elevator derivatives with the normalised thrust derivatives as given. Load "FDP Example 4-5(thrust).mat" to see this solution.

(v) Clearly, this process can be repeated as necessary and it is straightforward to convert the derivative data to another format if required. However, care is required to maintain visibility of the correct units applying – note that in this example American Imperial units apply ($g=32.2$ ft/s^2).

```
LONGITUDINAL REPORT FOR - FDP Example 4-5.mat

*-------------------*
*  xdot = Ax + Bu   *
*     y = Cx + Du   *
*-------------------*
u^T = [     Eta       ]
x^T = [     u         w          q        theta ]
Control Law :-
eta_d = delta - K [u]

K =

     0     0     0     0

STATE SPACE MATRICES :-

A =

       -0.00276        0.0389      -62.074        -32.096
      -0.065436      -0.31913       771.48        -2.5997
     0.00020059     -0.001013     -0.47949     0.00030157
              0             0            1              0


B_transposed =

         1.44        -18.02       -1.1579              0
```

---------- Full solution continues ----------

Example 4.6

FDA-CAD is used to show the process of deriving the lateral-directional state equation for an aircraft starting from the American normalised derivative format. Data for the same aircraft at the same flight condition as in Example 4.5 was used for this example and the data file should be loaded into FDA-CAD.

(i) Press the **'RUN MODEL'** button. This solves the equations of motion and in the process the lateral-directiona state equation is calculated.

(ii) Press the **'Print Report'** button. Open the Matlab Command Window to see the solution report. The state equation comprises the state matrix A and the input matrix B, which appear in the first part of the report as shown below.

(iii) Compare the A and B matrices with those in the equation given by equation (4.89) in Example 4.6.

(iv) Note that FDA-CAD has two inputs in the lateral-directional model - aileron and rudder. Observe that the B matrix has two columns to allow for two inputs.

(v) As in Example 4.5, note that again, American Imperial units apply (g=32.2 ft/s^2). Observe also that FDA-CAD is organised such the first of the four equations of motion in the state equation (the first row) represents sideslip angle β, and not sideslip velocity v. This is usually clearly stated in the output material.

LATERAL/DIRECTIONAL REPORT FOR - FDP Example 4-6.mat

K=[0 0 0 0 (Feedback gain matrix for augmented case equations)
0 0 0 0]

STATE SPACE MATRICES :-

A =

-0.055814	0.080199	-0.99678	0.041468
-3.05	-0.465	0.388	0
0.598	-0.0318	-0.115	0
0	1	0.080458	0

B_transposed =

0	0.143	0.00775	0	(Aileron input derivatives)
0.00729	0.153	-0.475	0	(Rudder input derivatives)

---------- Full solution continues ----------

Example 5.2

This example is intended to show the math steps in solving the equations of motion to obtain the response transfer functions. This process is, of course, hidden in FDA-CAD. However the computational solution can be found and compared with that obtained in the example. Load the model data file “FDP Example 5-2.mat” to FDA-CAD to see the flight condition and derivative data, which should be compared with that given in the example. It is important also to note the units applying, and these reflect the source of the information.

(i) Press the **‘RUN MODEL’** button. This solves the equations of motion and in the process the longitudinal state equation and a full set of response transfer functions are calculated.

(ii) Press the **'Print Report'** button. Open the Matlab Command Window to see the solution report. As before the state equation appears first and is followed by the full list of response transfer functions.

(iii) The transfer function of interest here is that describing pitch attitude response to elevator, this is the fourth transfer function to appear in the list and appears as follows,

```
TRANSFER FUNCTIONS - FACTORISED FORM

Zero/pole/gain from input to output "u - Axial velocity":
          -2.3668 (s+5.519) (s-4.215)
-----------------------------------------------------
(s^2  + 0.03326s + 0.02201) (s^2  + 0.8917s + 4.883)


Zero/pole/gain from input to output "w - Normal velocity":
   -22.1206 (s+64.67) (s^2  + 0.03485s + 0.02249)
-----------------------------------------------------
(s^2  + 0.03326s + 0.02201) (s^2  + 0.8917s + 4.883)


Zero/pole/gain from input to output "q - Pitch rate":
         -4.658 s (s+0.2688) (s+0.1335)
-----------------------------------------------------
(s^2  + 0.03326s + 0.02201) (s^2  + 0.8917s + 4.883)


Zero/pole/gain from input to output "theta - Pitch attitude":
          -4.658 (s+0.2688) (s+0.1335)
-----------------------------------------------------
(s^2  + 0.03326s + 0.02201) (s^2  + 0.8917s + 4.883)
```

---------- Full solution continues ----------

(iv) Comparison of the pitch attitude response transfer function with equation (5.24) in the example indicates a good match. Minor numerical differences are evident and these are almost certainly due to the accuracy of the hand calculation used in preparation of the FDP example. This is not a problem as the accuracy of either transfer function is more than adequate for flight dynamics analysis and for flight control system design. Note that the units of the input-output response described by the transfer function is rad/rad or, equivalently, deg/deg.

Example 5.6

This example can not be worked directly with FDA-CAD since it does not accept derivatives in the concise format. However, it does calculate the concise derivatives which appear as the coefficients in the A and B state matrices. Normalised aircraft data converted to SI units, see data file “FDP Example 5-6.mat”, were obtained from Heffley and Jewell[3] and input to FDA-CAD in the usual way. Push the **‘RUN MODEL’** button followed by the **‘Print Report’** button to obtain the solution of the equations of motion. It will be seen that the A and B matrices compare favourably with equation (5.58), with the exception of the concise derivative y_p. y_p was approximated by zero and this has disturbed some of the transfer functions as will be seen by comparing those obtained with FDA-CAD with those given by equations (5.61) and (5.62). Be aware also, that the sideslip transfer functions in the example refer to sideslip velocity, whereas those produce by FDA-CAD refer to sideslip angle. Dividing the gain of the sideslip velocity transfer functions in the example by aircraft velocity, converts the sideslip variable to sideslip angle, which compares directly with the transfer functions given in the FDA-CAD solution.

Example 6.1

This example shows the full solution of the longitudinal equations of motion and some typical response plots for the unaugmented aircraft. The aircraft model data should be loaded into FDA-CAD from file “FDP Example 6-1.mat”.

(i) Press the **‘RUN MODEL’** button. This solves the equations of motion and in the process the longitudinal state equation, a full set of response transfer functions and the longitudinal stability modes are calculated.

(ii) Press the **‘Print Report’** button. Open the Matlab Command Window to see the solution report. As before the state equation appears first, followed by the full list of response transfer functions and concludes with a summary of the stability modes characteristics. The full report is shown below and the transfer functions relevant to the example are highlighted. In particular, the state equation may be compared with equation (6.6) and the transfer functions with those given in equations (6.8)

```
LONGITUDINAL REPORT FOR - FDP Example 6-1.mat

*--------------------*
*  xdot = Ax + Bu   *
*     y = Cx + Du   *
*--------------------*
u^T = [     Eta       ]
x^T = [     u          w          q        theta ]
Control Law :-
eta_d = delta - K [u]

K =

     0     0     0     0
```

STATE SPACE MATRICES :-

A =

0.00501	0.00464	-72.926	-31.336
-0.0857	-0.545	308.5	-7.4076
0.0018453	-0.007673	-0.39491	0.0013186
0	0	1	0

B_transposed =

5.63	-23.8	-4.5158	0

C =

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1
-0.0857	-0.545	0	-7.4076
0	0.0031546	0	0
0	-0.0031546	0	1
0	-1	0	317

D_transposed =

0	0	0	0	-23.8	0	0	0

TRANSFER FUNCTIONS - FACTORISED FORM

```
Zero/pole/gain from input to output "u - Axial velocity":
       5.63 (s+58.46) (s+0.5865) (s+0.369)
-------------------------------------------------
(s^2  + 0.0332s + 0.01971) (s^2  + 0.9017s + 2.662)

Zero/pole/gain from input to output "w - Normal velocity":
   -23.8 (s+58.95) (s^2  - 0.008789s + 0.009798)
-------------------------------------------------
(s^2  + 0.0332s + 0.01971) (s^2  + 0.9017s + 2.662)

Zero/pole/gain from input to output "q - Pitch rate":
        -4.5158 s (s+0.5055) (s-0.008227)
-------------------------------------------------
(s^2  + 0.0332s + 0.01971) (s^2  + 0.9017s + 2.662)

Zero/pole/gain from input to output "theta - Pitch attitude":
         -4.5158 (s+0.5055) (s-0.008227)
-------------------------------------------------
(s^2  + 0.0332s + 0.01971) (s^2  + 0.9017s + 2.662)

Zero/pole/gain from input to output "Az - Normal acceleration":
     -23.8 s (s+5.664) (s-5.226) (s-0.02774)
-------------------------------------------------
(s^2  + 0.0332s + 0.01971) (s^2  + 0.9017s + 2.662)
```

```
Zero/pole/gain from input to output "alpha - Angle of attack":
 -0.075079 (s+58.95) (s^2  - 0.008789s + 0.009798)
---------------------------------------------------
(s^2  + 0.0332s + 0.01971) (s^2  + 0.9017s + 2.662)

Zero/pole/gain from input to output "gamma - Flight path angle":
    0.075079 (s-6.137) (s+4.961) (s-0.02719)
---------------------------------------------------
(s^2  + 0.0332s + 0.01971) (s^2  + 0.9017s + 2.662)

 Zero/pole/gain from input to output "dh - height rate":
      23.8 (s-6.137) (s+4.961) (s-0.02719)
---------------------------------------------------
(s^2  + 0.0332s + 0.01971) (s^2  + 0.9017s + 2.662)
```

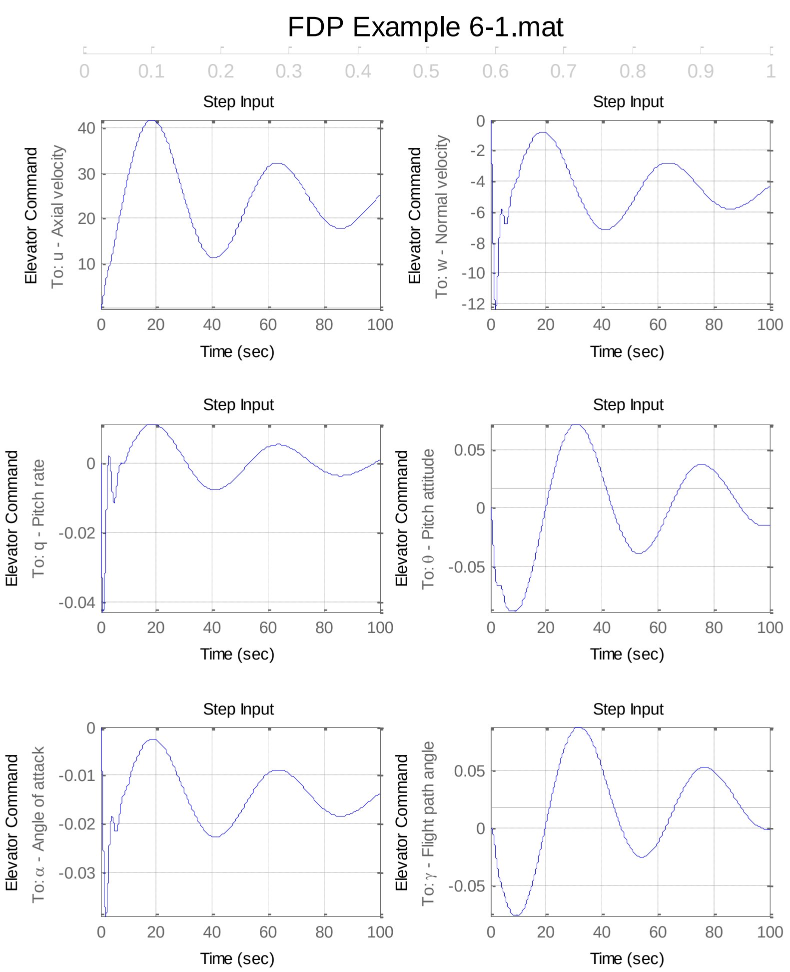
CHARACTERISTIC POLE LOCATIONS

Eigenvalue	Damping	Freq. (rad/s)	
-1.66e-002 + 1.39e-001i	1.18e-001	1.40e-001	(Phugoid mode damping and frequency)
-1.66e-002 - 1.39e-001i	1.18e-001	1.40e-001	
-4.51e-001 + 1.57e+000i	2.76e-001	1.63e+000	(Short period mode damping and frequency)
-4.51e-001 - 1.57e+000i	2.76e-001	1.63e+000	

---------- End of solution report ----------

(iii) Compare the stability mode characteristics given above with those discussed in Example 6.1.

(iv) To use FDA-CAD to reproduce the response plots shown in Fig.6.1, first check only those output variables required in the **'Output Responses'** window in FDA-CAD. Select the **'Long Period'** output response time (100s) and remember to set the **'Input Signal'** to a step of magnitude 0.075rad (1 deg). Push the **'Plot Responses'**



button to open the figure window with the annotated response time histories as selected. A copy of the figure window is shown above.

(v) If wider plots are preferred to match those shown in Fig.6.1, then FDA-CAD should be set to plot no more than four variables at one time – i.e. two figures.

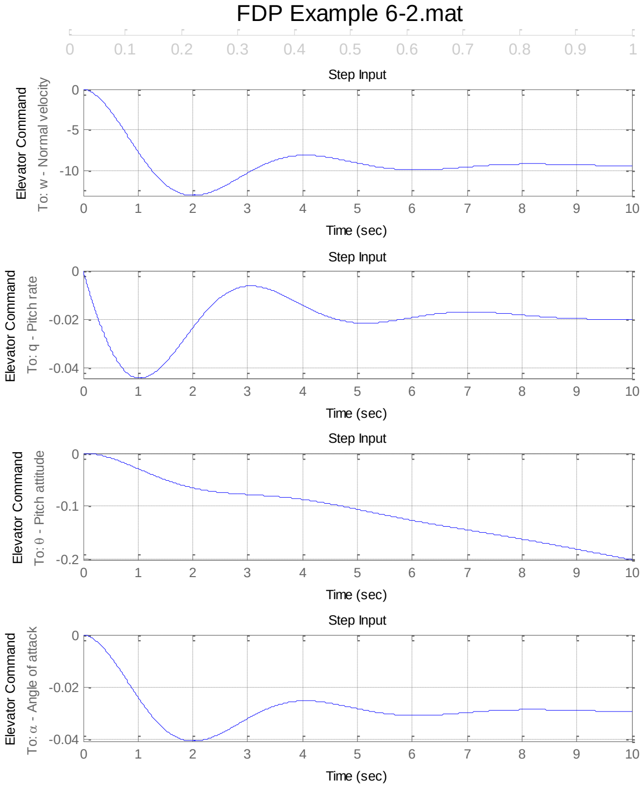
(vi) The plots can be re-scaled and additional information can be added using the standard Matlab tools in the figure window prior to saving or printing.

Example 6.2

This example shows the derivation and solution of the longitudinal *reduced order* equations of motion, together with some typical response plots for the unaugmented aircraft. The aircraft model is the same as that used in Example 6.1.

(i) With the model data loaded into FDA-CAD select '**Reduced Order'** in the **'Model Dynamics'** pop-up menu. Push the **'RUN MODEL'** button followed by the **'Print Report'** button. Compare the reduced order report with that obtained for Example 6.1.

(ii) The model is now third order – second order is shown in FDP Example 6.1. FDA-CAD retains pitch attitude in the model in order to remain compatible with both a wind axes reference and a body axes reference. Reduced order pitch attitude response is also useful in handling qualities assessment.



(iii) Set up the response plotting requirement for variables w, q, θ, α, select the response time for 10s and select the input to be a 1deg step. Push the **'Plot Responses'** button to open the figure window showing the four time histories. A copy of the figure window is shown, and the responses should be compared with the those shown in FDP Fig.6.5. Unfortunately, multiple comparative plots on the same axes cannot be made from the basic aircraft screen in this version of FDA-CAD.

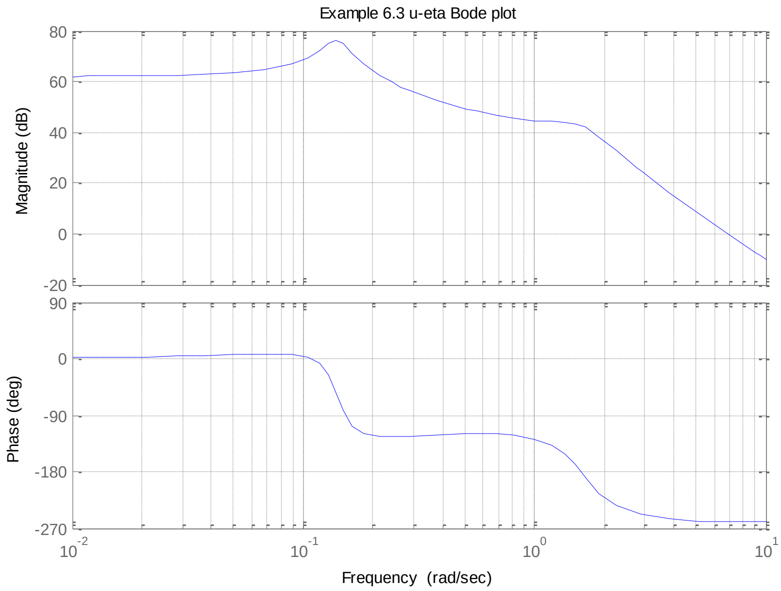
Example 6.3

This example illustrates the use of FDA-CAD to obtain longitudinal Bode frequency response plots from the basic aircraft transfer functions. The aircraft model is the same as that used in Example 6.1, but the derivative data is first converted to a wind axes reference.

(i) With the model data for Example 6.1 loaded into FDA-CAD, convert the derivative data to a wind axes reference and obtain the following report.

```
=========================================================
DERIVATIVE CONVERSION REPORT FOR: FDP Example 6-1.mat
=========================================================

General Flight Conditions:
Mach: 0.30
Height: 15000 ft
Body Incidence: 13.30 deg

Original Axes notation: BODY AXES

=======Normalised [USA] Derivatives in Body Axes=======

Xu=     0.00501      Zu=    -0.08570      Mu=     0.00183
Xw=     0.00464      Zw=    -0.54500      Mw=    -0.00777
Xdw=    0.00000      Zdw=    0.00000      Mdw=   -0.00018
Xq=     0.00000      Zq=     0.00000      Mq=    -0.34000
Xeta=   5.63000      Zeta=  -23.80000     Meta=  -4.52000

=======Normalised [USA] Derivatives in Wind Axes=======

Xu=    -0.04225      Zu=    -0.20455      Mu=    -0.00001
Xw=    -0.11421      Zw=    -0.49774      Mw=    -0.00798
Xdw=    0.00000      Zdw=    0.00000      Mdw=   -0.00017
Xq=     0.00000      Zq=     0.00000      Mq=    -0.34000
Xeta=   0.00381      Zeta=  -24.45684     Meta=  -4.52000
```

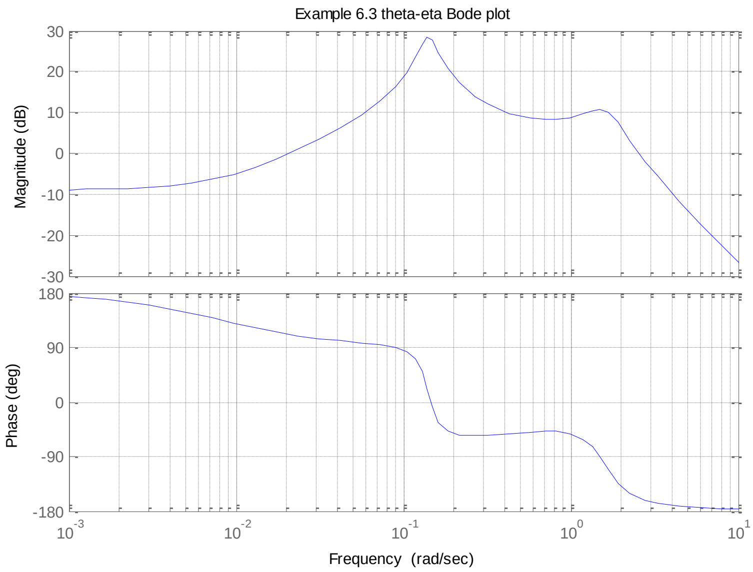
(ii) Key in the wind axes referenced derivatives and change the body incidence value to 0deg in the flight data field. Change the aircraft axes to wind axes in the pop-up menu, then push the **'Run Model'** button followed by the **'Print Report'** button. Compare the report with that obtained for Example 6.1 and note any differences.



(iii) Push the yellow **'u'** button in the output window to open the Bode plot figure window for that variable. To facilitate comparison with Fig.6.7, it is convenient to use the Matlab tools to change the scales of the Bode gain and phase plots. A print out from the figure window is shown and it will be seen to compare favourably with Fig.6.7.

(iv) Repeat the process to obtain a Bode plot of pitch attitude in the figure window. Again, to facilitate comparison with Fig.6.8, use the Matlab tools to change the scales of the Bode gain and phase plots. A print out from the figure window is shown below and it will be seen to compare favourably with Fig.6.8, with the exception of the phase values. Low frequency phase should be zero, but as the transfer function governing pitch attitude in this example is non-minimum phase, the computational solution interprets phase as shown. Care is needed to be aware that many aircraft transfer functions are non-minimum phase, which may produce unexpected phase frequency response. Further, it is common to find that typical software tools differ in the way that non-minimum phase transfer functions are interpreted.

(v) It is left as an exercise for the user to obtain the Bode plot for alpha α response to elevator η. Adjust the scales of the Bode plot obtained to match the plots shown in Fig.6.9.



Example 7.1

This example shows the full solution of the lateral-directional equations of motion and some typical response plots for the unaugmented aircraft. The aircraft model data should be loaded into FDA-CAD from file “FDP Example 7-1.mat”.

(i) Press the **‘RUN MODEL’** button. This solves the equations of motion and in the process the lateral-directional state equation, a full set of response transfer functions and the longitudinal stability modes are calculated.

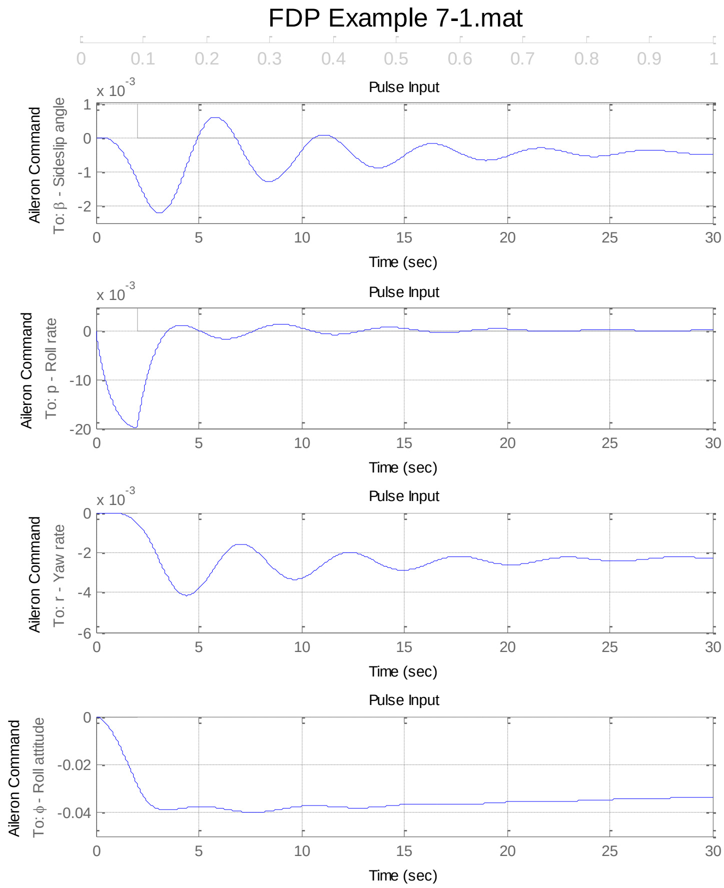
(ii) Press the **‘Print Report’** button. Open the Matlab Command Window to see the solution report. As before, the state equation appears first, followed by the full list of response transfer functions and concludes with a summary of the stability modes characteristics. The full report is shown below. In particular the state equation, matrices A and B, may be compared with equation (7.10). The transfer functions describing response to aileron compare with those given in equations (7.12) and the transfer functions describing response to rudder compare with those given in equations (7.13). The characteristic pole locations given by FDA-CAD compare with equation (7.14).

```
LATERAL/DIRECTIONAL REPORT FOR - FDP Example 7-1.mat

K=[0 0 0 0
   0 0 0 0]
STATE SPACE MATRICES :-

A =

      -0.1008            0       -468.2        32.2
   -0.0057881       -1.232        0.397           0
    0.0027787      -0.0346       -0.257           0
            0            1            0           0


B_transposed =

            0        -1.62     -0.01875           0     (Aileron input derivatives)
       13.484        0.392       -0.864           0     (Rudder input derivatives)


C =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

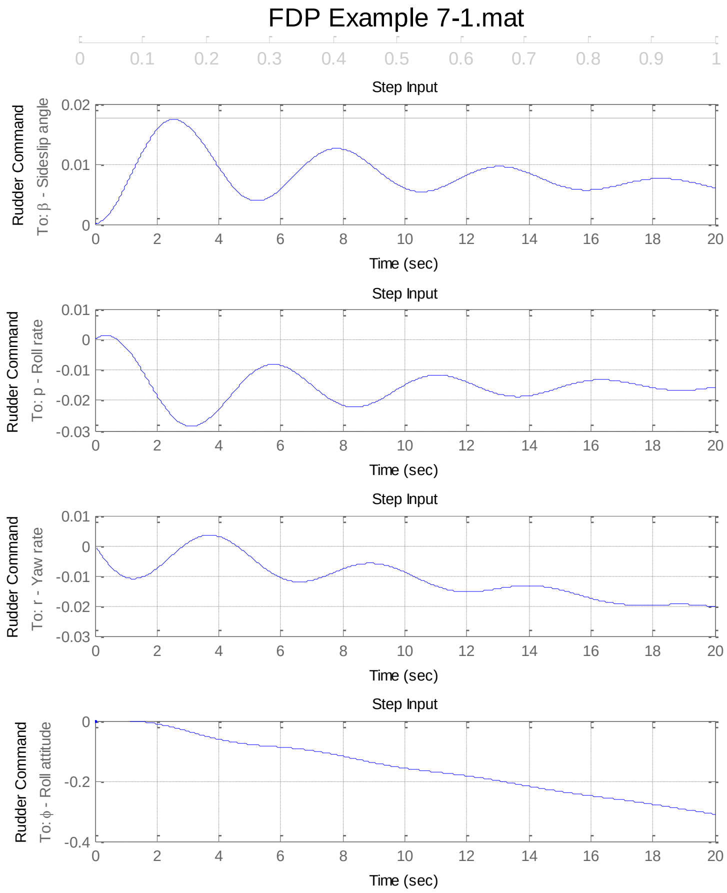
```
D_transposed =

     0     0     0     0
     0     0     0     0

TRANSFER FUNCTIONS - FACTORISED FORM
 (Transfer functions describing response to aileron)
Zero/pole/gain from input to output "beta A - Sideslip due to Aileron":
         0.01875 (s-7.896) (s+0.1969)
----------------------------------------------
(s+1.329) (s+0.006498) (s^2  + 0.2543s + 1.433)

 Zero/pole/gain from input to output "pA - Roll rate due to Aileron":
       -1.62 s (s^2  + 0.3624s + 1.359)
----------------------------------------------
(s+1.329) (s+0.006498) (s^2  + 0.2543s + 1.433)

Zero/pole/gain from input to output "rA - Yaw rate due to Aileron":
  -0.01875 (s+1.589) (s^2  - 3.246s + 4.982)
----------------------------------------------
(s+1.329) (s+0.006498) (s^2  + 0.2543s + 1.433)

Zero/pole/gain from input to output "phi A - Roll attitude due to Aileron":
        -1.62 (s^2  + 0.3624s + 1.359)
----------------------------------------------
(s+1.329) (s+0.006498) (s^2  + 0.2543s + 1.433)

(Transfer functions describing response to rudder)
Zero/pole/gain from input to output "beta R - Sideslip due to Rudder":
    0.0288 (s+30.21) (s+1.296) (s-0.01477)
----------------------------------------------
(s+1.329) (s+0.006498) (s^2  + 0.2543s + 1.433)

Zero/pole/gain from input to output "pR - Roll rate due to Rudder":
          0.392 s (s-2.566) (s+1.85)
----------------------------------------------
(s+1.329) (s+0.006498) (s^2  + 0.2543s + 1.433)

Zero/pole/gain from input to output "rR - Yaw rate due to Rudder":
  -0.864 (s+1.335) (s^2  - 0.02995s + 0.1092)
----------------------------------------------
(s+1.329) (s+0.006498) (s^2  + 0.2543s + 1.433)

Zero/pole/gain from input to output "phi R - Roll attitude due to Rudder":
           0.392 (s-2.566) (s+1.85)
----------------------------------------------
(s+1.329) (s+0.006498) (s^2  + 0.2543s + 1.433)

CHARACTERISTIC POLE LOCATIONS

        Eigenvalue                  Damping      Freq. (rad/s)

 -6.50e-003                   1.00e+000     6.50e-003
 -1.27e-001 + 1.19e+000i     1.06e-001     1.20e+000
 -1.27e-001 - 1.19e+000i     1.06e-001     1.20e+000
 -1.33e+000                   1.00e+000     1.33e+000
```

(iii) Note that FDA-CAD shows the sideslip response transfer functions in terms of sideslip angle β only. To obtain the transfer functions in terms of sideslip velocity v it necessary only to multiply those in terms of β with aircraft velocity V. It follows that v then has the units of velocity appropriate to the aircraft model data.

(iv) FDA-CAD may now be used to reproduce the response plots shown in Fig.7.1 and in Fig.7.2. First, to obtain a set of responses to aileron to match Fig.7.1, uncheck the response to rudder variables in the outputs window. Enter a response time of 30s in the **'Select Response Time'** field and set the **'Input Signal'** to a pulse of magnitude 0.075rad (1 deg) and a width of 2.0s. Push the **'Plot Responses'** button to open the figure window with the annotated response time histories as selected. A copy of the figure window is shown here. Note that the y-axis scales have been adjusted to exactly match those shown in Fig.7.1 – this is done by opening the properties editor for each plot in turn within the Matlab figure window and editing the scale limits.



(v) Second, to obtain a set of responses to rudder to match Fig.7.2, check the response to rudder variables and uncheck the response to aileron variables in the outputs window. Enter a response time of 20s in the **'Select Response Time'** window and set the **'Input Signal'** to a step of magnitude 0.075rad (1 deg). Push the **'Plot Responses'** button to open the figure window with the annotated response time histories as selected. A copy of the figure window is shown here. Again, the y-axis scales have been adjusted to exactly match those shown in Fig.7.2.



(vi) The plots can be re-scaled and additional information can be added using the standard Matlab tools in the figure window prior to saving or printing. Choice of scales for the plots is important if the adverse response properties are not to be missed. Such properties are the result of non-minimum phase transfer functions and care should be exercised when conducting analysis with FDA-CAD to correlate the visible response "shapes" with their describing transfer functions.

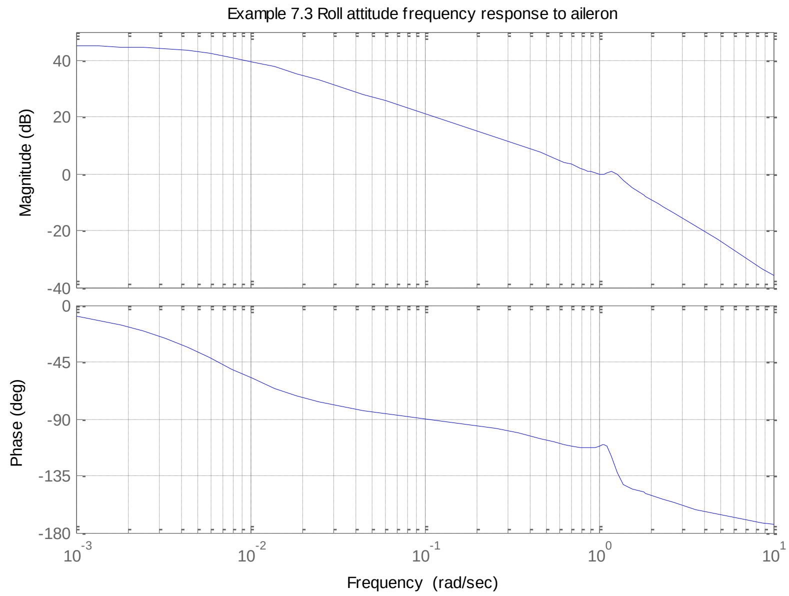
Example 7.3

This example illustrates the use of FDA-CAD to obtain lateral-directional Bode frequency response plots from the basic aircraft transfer functions. The aircraft model is the same as that used in Example 7.1.

(i) With the model data loaded to FDA-CAD, push the **'RUN MODEL'** button followed by the **'Print Report'** button. Retain the report for reference to the transfer functions of interest.

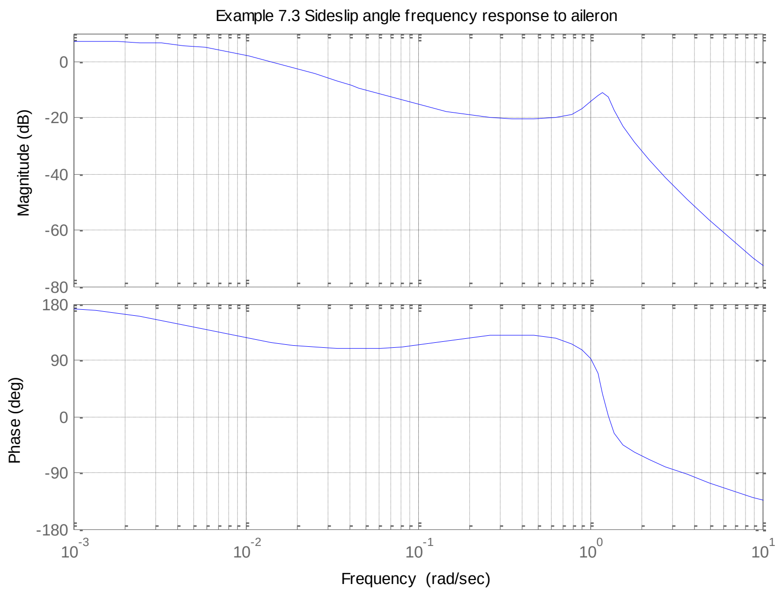
(ii) Push the yellow **'ϕ'** button in the *aileron* column of the output window to open the Bode plot figure window for that variable. The Bode plot compares directly with Fig.7.7. Use the Matlab tools to change the scales of the Bode gain and phase plots to match those of Fig.7.7. A print out from the figure window is shown below and it will be seen to compare favourably with Fig.7.7.

(iii) Repeat the process by pushing the yellow **'β'** button in the *aileron* column of the output window to open the Bode plot figure window for that variable. As before, use the Matlab tools to change the scales of the Bode gain and phase plots to match those of Fig.7.8. A print out from the figure window is shown below and it will be seen to compare favourably with Fig.7.8.



(iv) Note the difference in phase scale with that shown in Fig.7.8. This is due to the way in which different computer programs deal with non-minimum phase transfer functions – Fig.7.8 was obtained with the aid of Program CC. This problem of Bode frequency interpretation is discussed in FDP.

(v) It is left as an exercise for the user to obtain the frequency response plots shown in Fig.7.9 and in Fig.7.10. Adverse response to rudder is an established feature of aircraft dynamics, and the consequent non-minimum phase effects



appear in the Bode plots. It is sufficient that the user is aware of these properties when interpreting the output of analysis using FDA-CAD.

Example 11.4

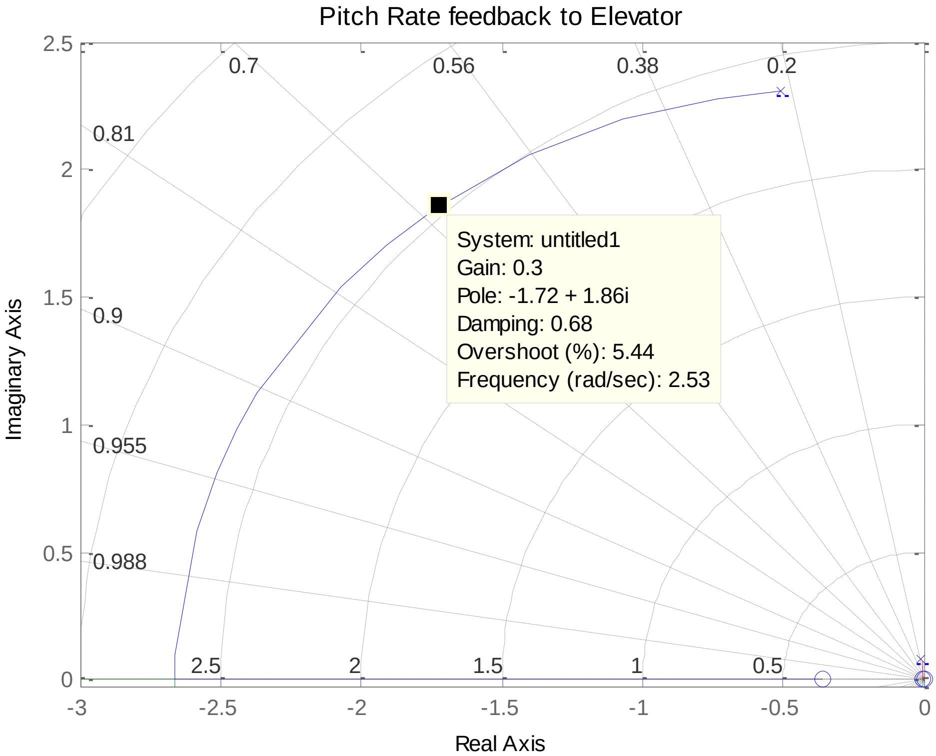
This example is intended to show the effect of single output variable feedback to elevator on the longitudinal dynamics of an aircraft. The root locus plot is used for this purpose and FDA-CAD enables the analysis to be made quickly and easily. It is first necessary to load the aircraft model data file “FDP Example 11-4.mat”.

(i) Push the **‘RUN MODEL’** button followed by the **‘Print Report’** button to obtain an overview of the basic aircraft transfer functions and also the stability modes characteristics. If response plots for the unaugmented aircraft are required they should also be obtained at this time, and the process for so doing has been discussed in earlier examples. It is useful to obtain hard copy print output showing the basic aircraft properties as this information is useful for reference during the design of closed loop feedback gains.

(ii) Once the analysis of the basic unaugmented aircraft is complete press the **‘Stability and Control Augmentation’** button. This opens a new screen showing the longitudinal feedback structure model in FDA-CAD. Note that alternative structures are not possible. The user can switch between screens as necessary during the course of an analytical study.

(iii) To investigate pitch rate feedback to elevator and re-create Fig.11.13, push the yellow **‘Kq’** button in the feedback structure window. This produces the root locus plot in the reserved plot window which may be interrogated by the user, using the standard Matlab tools, to evaluate the effect of q feedback on the stability properties of the aircraft. The feedback gain to achieve a chosen mode frequency and damping ratio can be identified by moving the mouse cursor along the appropriate locus. A reminder in the top left of the plot window states whether the feedback gain should be entered into the model with a positive or negative sign. To obtain a hard copy record of the root locus plot, click on the **‘Root Locus Plot’** button in the top left screen tool bar, and then click on the **‘undock’** button to open the usual Matlab figure window. The plot may be edited as required and annotated with gain test points as desired. For this illustration the plot was rescaled twice, once to show the short period mode dynamics clearly and once to show the phugoid dynamics clearly. Copies of both plots were obtained as shown below.

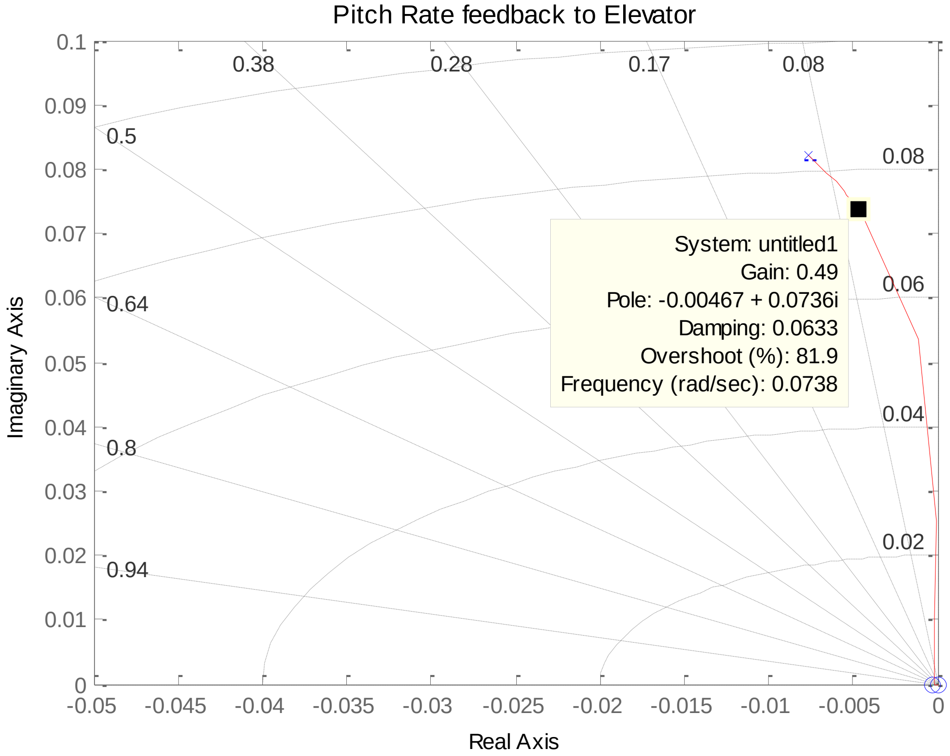
Large scale plot to show short period dynamics Fig 11.13



(iv) This procedure can be repeated for all of the variables discussed in FDP Example 11.4 which is left as an exercise for the user. In a typical system design analysis, the feedback gain requirements can usually be established in the reserved screen plot window. It is only generally necessary to undock the figure when a printed copy of the conclusion is required for the record.

(v) To obtain the closed loop equations of motion, enter the chosen feedback gain value (K_q=-0.3) in the **'Kq'** field in the feedback structure window, push the **'RUN MODEL'** button and observe the change in the stability modes in the **'Aircraft Modes'** window. Push the **'Print Report'** button to see the entire solution in the Matlab Command Window. At any time, the user can see a reminder of the basic aircraft state equation by clicking on the **'Basic Aircraft'** box in the system structure window.

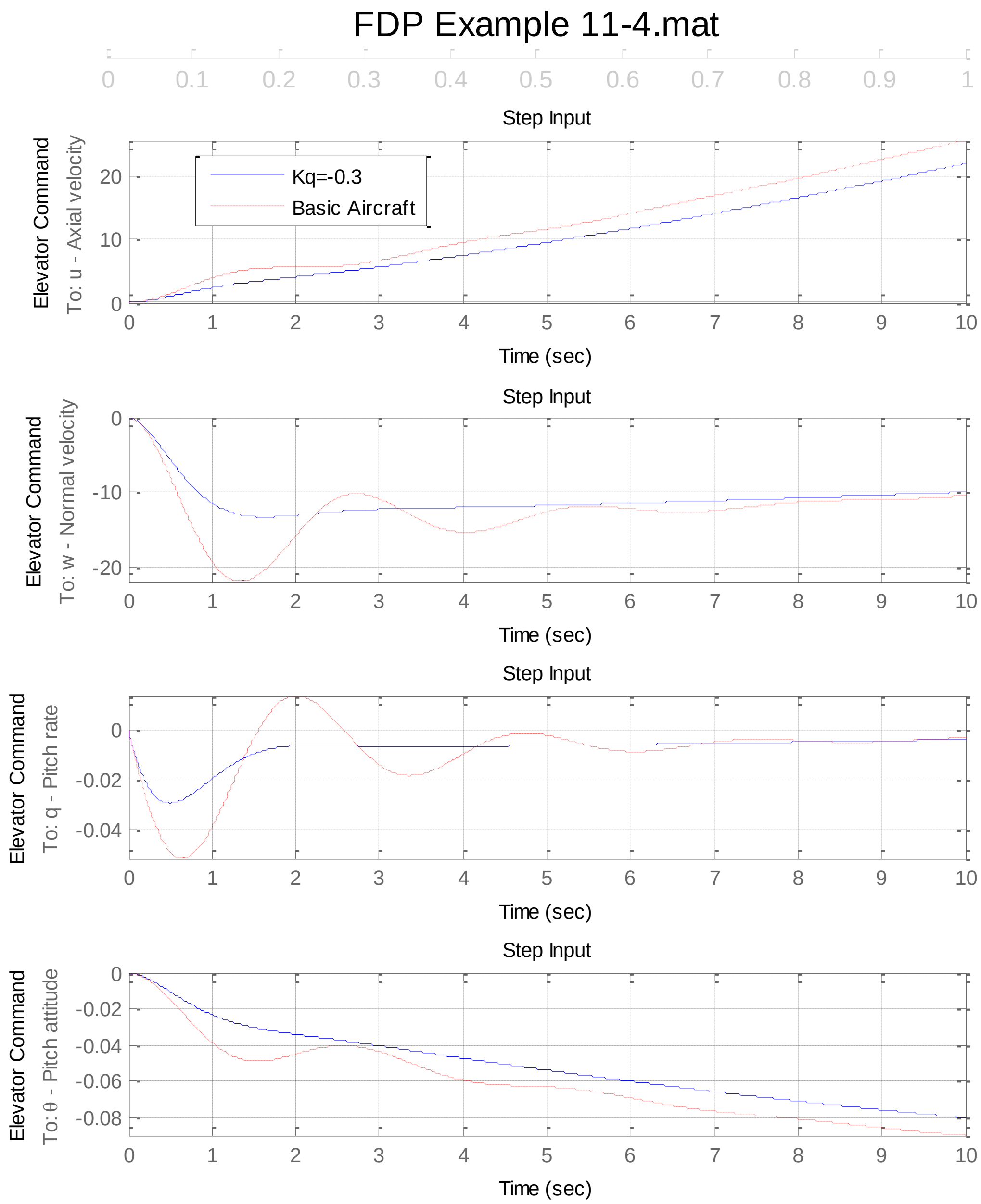
Small scale plot to show phugoid dynamics Fig 11.13



(vi) Having designed a typical feedback loop and having calculated the closed loop system model, the user can now design an additional feedback loop by the same process. This can be repeated for as many loop closures up to the maximum of four. For example, a statically unstable aircraft typically requires both α and q feedback to achieve a satisfactory solution. In such an example the gain K_{α} would be designed first to restore the static margin to the desired value. With the chosen value of K_{α} entered into the appropriate feedback gain field push the **'RUN MODEL'** button to obtain the closed loop equations of motion with the α loop closed. Using this closed loop model FDA-CAD can now be used to design a value for K_q to restore the pitch damping to an acceptable value as described above. Entering the chosen value of feedback gain into the Kq field and pushing the **'RUN MODEL'** button produces the equations of motion with both feedback loops closed. Push the **'Print Report'** button to see the full solution.

(vii) At any time during the analysis the user may add an actuator to the system model. The actuator transfer function numerator and denominator are entered into the actuator window fields and pushing the **'RUN MODEL'** button updates the closed loop system model accordingly. The actuator mode then appears in the **'Aircraft Modes'** window. With no feedback gains in the model, the solution is then simply the basic aircraft augmented to include the open loop actuator dynamics.

(viii) Response time histories can be obtained at any time during the development of a feedback gain structure and FDA-CAD enables two sets of time histories to be compared to illustrate the effects of feedback and other system changes. In the present example, compare the basic aircraft with the augmented aircraft having pitch rate feedback to elevator. Select a step **'Input Signal'** of one degree magnitude (0.0175) and with no feedback gains set in the feedback gain fields, push the **'RUN MODEL'** button to obtain the basic aircraft solution. Check the four variables u, w, q and θ in the **'Output'** window, select **'Short Period'** response time (10s) and push the **'Plot Responses'** button. The plots will appear in a Matlab figure window as described before. Toggle the **'Save Plot 1'** button, the data is saved and the window is closed. Now enter K_q=-0.3 in the appropriate feedback gain field and push **'RUN MODEL'** to obtain the closed loop equations of motion. Repeat the plot process for the same input to see the closed loop time histories in the figure window. Toggle the **'Save Plot 2'** button to save the plot data and close the window. With both save buttons still toggled, push the **'Compare Plots 1 and 2'** button. The Matlab figure window opens to show both sets of plots on the same axes as shown below. Note that the plots legend has been edited to suit.



Example 11.5

With reference to FDP, Example 11.5 is the lateral-directional equivalent of longitudinal Example 11.4. Aircraft model data will be found in data file “FDP Example 11-5.mat”, and its analysis follows the procedures described in the previous example. However, there are some minor differences.

(i) The user must toggle between the lateral feedback structure window and the directional feedback structure window by means of the pop-up menu alongside the file name window at the top of the screen. The **‘RUN MODEL’** function includes both lateral feedback to aileron and directional feedback to rudder in its solution. The model also has provision for an aileron actuator, a rudder actuator and an aileron-rudder interlink gain. In all other respects interpretation of the functional tools in the GUI screen is similar to previous examples.

(ii) It is left as an exercise for the user to investigate the lateral-directional feedback design capabilities of FDA-CAD.

Example 11.6

The purpose of this example is to show the pole placement capability of FDA-CAD in the design of a simple full state feedback matrix. Aircraft model data will be found in the file "FDP Example 11-6.mat", which should loaded for longitudinal analysis in FDA-CAD. Note that the data have been transferred to SI units throughout.

(i) Push the **'RUN MODEL'** button followed by the **'Print Report'** button to obtain an overview of the basic aircraft state equation and also the stability modes characteristics. Extracts from the report required for the example are shown below.

```
LONGITUDINAL REPORT FOR - FDP Example 11-6.mat

*-------------------*
*  xdot = Ax + Bu   *
*     y = Cx + Du   *
*-------------------*
u^T = [     Eta       ]
x^T = [     u          w          q        theta ]
Control Law :-
eta_d = delta - K [u]

K =

     0     0     0     0

STATE SPACE MATRICES :-   (Basic unaugmented aircraft.)

A =

      -0.0677      -0.0107      1.9635          -9.8099
    0.022527       -2.1031      371.13         0.051198
    0.010736      -0.15507     -3.0877     -0.00012247
           0             0           1                0


B_transposed =

     -0.4023      -76.257      -60.918          0
```

C =

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1
0.022527	-2.1031	-3.8677	-0.00016691
0	0.0026667	0	0
0	-0.0026667	0	1
0	-1	0	375

D_transposed =

0	0	0	0	-76.257	0	0	0

----------------- Continues to: -----------------

CHARACTERISTIC POLE LOCATIONS

Eigenvalue	Damping	Freq. (rad/s)
-3.47e-002 + 4.16e-002i	6.40e-001	5.42e-002
-3.47e-002 - 4.16e-002i	6.40e-001	5.42e-002
-2.59e+000 + 7.57e+000i	3.24e-001	8.00e+000
-2.59e+000 - 7.57e+000i	3.24e-001	8.00e+000

(ii) Compare the State matrices A and B with those given in Example 11.6, and compare the stability modes characteristics also. Note some numerical differences due to the fact that the example was originally worked by hand and some approximations were made – terms A(2,4) and A(3,4) were both approximated by zero, and A(3,3), the pitch damping term, was approximated by the stability derivative m_q. The last approximation causes an error in the value of the short period mode damping, but this does not significantly change the interpretation of aircraft dynamics. The solution obtained with FDA-CAD matches closely that given in the source reference Heffley and Jewell[3].

(iii) Switch to the **'Stability and Control Augmentation'** screen. Observe that the basic aircraft short period mode and phugoid mode characteristics are shown in the **'Pole Placement'** window fields for reference. The same information is also shown in the '**'Aircraft Modes'** window below.

(iv) Choose the desired new values for short period and phugoid mode frequency and damping as explained in the example. Enter these values in the **'Pole Placement'** window fields. Push the **'Pole Placement'** button and the feedback gain values required to achieve these modes characteristics will appear in the feedback gain fields in the system structure window. Push the **‘RUN MODEL’** button to obtain the solution of the closed loop equations of motion. Push the ‘**Print Report’** button to obtain the closed loop solution report in the Matlab Command Window as shown below. Observe that as full state feedback is used the stability modes characteristics are exactly as specified.

```
LONGITUDINAL REPORT FOR - FDP Example 11-6.mat

*-------------------*
*  xdot = Ax + Bu   *
*     y = Cx + Du   *
*-------------------*
u^T = [    Eta       ]
x^T = [    u          w          q        theta ]
Control Law :-
eta_d = delta - K [u]

K =

 -7.8693e-006   0.00050711    -0.099319   -0.0006542              (Feedback gain matrix)

STATE SPACE MATRICES :-

A =                       (Closd loop state matrix)

   -0.067703   -0.010496       1.9235       -9.8101
    0.021926       -2.0645      363.55    0.0013106
    0.010257     -0.12418       -9.138    -0.039974
           0             0            1            0

B_transposed =         (Closed loop input matrix)

     -0.4023      -76.257      -60.918          0


C =

          1          0          0          0
          0          1          0          0
          0          0          1          0
          0          0          0          1
    0.021926          -2.0645      -11.442    0.0013106
          0          0.0026667           0            0
          0         -0.0026667           0            1
          0                 -1           0          375

D_transposed =

    0    0    0    0    0    0    0    0
```

------------- Continues to: -------------------

CHARACTERISTIC POLE LOCATIONS (Closed loop stability modes)

Eigenvalue	Damping	Freq. (rad/s)
-3.51e-002 + 4.10e-002i	6.50e-001	5.40e-002
-3.51e-002 - 4.10e-002i	6.50e-001	5.40e-002
-5.60e+000 + 5.71e+000i	7.00e-001	8.00e+000
-5.60e+000 - 5.71e+000i	7.00e-001	8.00e+000

(v) The feedback system design can be simplified by observing that the gains K_u, K_w and K_θ are sufficiently small that they can be safely approximated by zero. Accordingly, enter a value of zero in the feedback gain fields. Choose also to round the pitch rate feedback gain to K_q=-0.1, and this value should be entered in the appropriate feedback gain field. Push the **'RUN MODEL'** button to solve the revised closed loop equations of motion. Push the **'Print Report'** button and obtain the revised closed loop solution report as shown below.

LONGITUDINAL REPORT FOR - FDP Example 11-6.mat

```
*------------------*
*  xdot = Ax + Bu   *
*     y = Cx + Du   *
*------------------*
u^T = [     Eta        ]
x^T = [     u          w          q        theta ]
Control Law :-
eta_d = delta - K [u]
```

K =

0 0 -0.1 0 (Feedback gain matrix showing single gain value.)

STATE SPACE MATRICES :-

A =

-0.067703	-0.0107	1.9233	-9.8099
0.021926	-2.1031	363.5	0.051198
0.010257	-0.15507	-9.1795	-0.00012247
0	0	1	0

B_transposed =

-0.4023	-76.257	-60.918	0

C =

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

0.021926	-2.1031	-11.493	0.051198
0	0.0026667	0	0
0	-0.0026667	0	1
0	-1	0	375

D_transposed =

0 0 0 0 0 0 0 0

TRANSFER FUNCTIONS - FACTORISED FORM

Zero/pole/gain from input to output "u - Axial velocity":

-0.4023 (s+305.2) (s-6.245) (s+1.488)

(s^2 + 0.06891s + 0.002364) (s^2 + 11.28s + 75.64)

Zero/pole/gain from input to output "w - Normal velocity":

-76.2574 (s+299.6) (s+0.07117) (s-0.00321)

(s^2 + 0.06891s + 0.002364) (s^2 + 11.28s + 75.64)

Zero/pole/gain from input to output "q - Pitch rate":

-60.9176 s (s+1.909) (s+0.06782)

(s^2 + 0.06891s + 0.002364) (s^2 + 11.28s + 75.64)

Zero/pole/gain from input to output "theta - Pitch attitude":

-60.9176 (s+1.909) (s+0.06782)

(s^2 + 0.06891s + 0.002364) (s^2 + 11.28s + 75.64)

-------------- Continues to: ----------------------

CHARACTERISTIC POLE LOCATIONS

Eigenvalue	Damping	Freq. (rad/s)
-3.45e-002 + 3.43e-002i	7.09e-001	4.86e-002
-3.45e-002 - 3.43e-002i	7.09e-001	4.86e-002
-5.64e+000 + 6.62e+000i	6.49e-001	8.70e+000
-5.64e+000 - 6.62e+000i	6.49e-001	8.70e+000

(vi) It will be seen that the closed loop transfer functions are a good match with those given in Example 11.6, equations (11.65) and (11.66), with the exception of the numerator of the velocity u response to elevator transfer function. This is though to be due to the different solution algorithms used in the software to work the example. Because aircraft matrix equations are not always well conditioned, the solutions do not always converge on the correct numerical coefficient values. However, in such cases the time response plots are usually correct.

6 REFERENCES

[1] Cook, M.V. 2012. ***Flight Dynamics Principles***, 3rd Edition. Elsevier, Ltd. Oxford, UK.

[2] Teper, G. L. 1969: ***Aircraft Stability and Control Data***. Systems Technology, Inc., STI Technical Report 176-1.

[3] Heffley, R. K. and Jewell, W. F. 1972: ***Aircraft Handling Qualities Data***. NASA Contractor Report, NASA CR-2144.

7 NOTATION

Standard symbols

a or A	Acceleration
A	State matrix
b	Wing-span
B	Input matrix
$\bar{\bar{c}}$	Mean aerodynamic chord (mac)
C	Output matrix
D	Direct matrix
f	force
g	Acceleration due to gravity
h	Height
I_x	Moment of inertia in roll
I_y	Moment of inertia in pitch
I_z	Moment of inertia in yaw
I_{xz}	Inertia product about *ox* and *oz* axes
K	Feedback gain
K	Feedback gain matrix
L	Rolling moment
m	Mass
M	Pitching moment
M	Mass matrix
N	Yawing moment
p	Roll rate
q	Pitch rate
r	Yaw rate
S	Wing reference area
t	Time
u	Axial velocity
U_e	Axial component of steady equilibrium velocity
v	Lateral velocity
V_0	Steady equilibrium velocity
w	Normal velocity
W_e	Normal component of steady equilibrium velocity
x	Axial displacement
x	State vector
X	Axial force
y	Lateral displacement
y	Output vector
Y	Lateral force
z	Normal displacement
Z	Normal force

Greek symbols

α	Angle of attack, Body incidence
β	Sideslip angle
γ	Flight path angle
δ	Control angle
δ_ξ	Roll control stick angle
δ_η	Pitch control stick angle
δ_ζ	Yaw control stick angle
ζ	Rudder angle
ζ_d	Dutch roll damping ratio
ζ_p	Phugoid damping ratio
ζ_s	Short period pitching oscillation damping ratio
η	Elevator angle
θ	Pitch angle
ξ	Aileron angle
ρ	Air density
ϕ	Roll angle
ψ	Yaw angle
ω	Undamped natural frequency

Subscripts

d	Dutch roll
e	Equilibrium
l	Rolling moment
m	Pitching moment
n	Yawing moment
p	Phugoid
p	Roll rate
q	Pitch rate
r	Roll mode
r	Yaw rate
s	Short period pitching oscillation. Spiral Mode
u	Axial velocity
v	Lateral velocity
w	Normal velocity
x	*ox* axis
y	*oy* axis
z	*oz* axis
α	Angle of attack or incidence
ζ	Rudder
η	Elevator
θ	Pitch
ξ	Aileron

Stability Derivatives

Notation	FDACAD	Description
X_u	Xu	Axial velocity due to velocity
X_w	Xw	Axial velocity due to incidence
$X_{\dot{w}}$	Xdw	Axial velocity due to downwash lag
X_q	Xq	Axial velocity due to pitch rate
$X_{\delta e}$	Xde	Axial velocity due to elevator
Z_u	Zu	Normal velocity due to velocity
Z_w	Zw	Normal velocity due to incidence
$Z_{\dot{w}}$	Zdw	Normal velocity due to downwash lag
Z_q	Zq	Normal velocity due to pitch rate
$Z_{\delta e}$	Zde	Normal velocity due to elevator
M_u	Mu	Pitching moment due to velocity
M_w	Mw	Pitching moment due to incidence
$M_{\dot{w}}$	Mdw	Pitching moment due to downwash lag
M_q	Mq	Pitching moment due to pitch rate
$M_{\delta e}$	Mde	Pitching moment due to elevator
Y_v	Yv	Side force due to lateral velocity
Y_p	Yp	Side force due to roll rate
Y_r	Yr	Side force due to yaw rate
$Y_{\delta a}$	Yda	Side force due to aileron
$Y_{\delta r}$	Ydr	Side force due to rudder
L_v	Lv	Rolling moment due to lateral velocity
L_p	Lp	Rolling moment due to roll rate
L_r	Lr	Rolling moment due to yaw rate
$L_{\delta a}$	Lda	Rolling moment due to aileron
$L_{\delta r}$	Ldr	Rolling moment due to rudder
N_v	Nv	Yawing moment due to lateral velocity
N_p	Np	Yawing moment due to roll rate
N_r	Nr	Yawing moment due to yaw rate
$N_{\delta a}$	Nda	Yawing moment due to aileron
$N_{\delta r}$	Ndr	Yawing moment due to rudder

Abbreviations

FDA-CAD	Flight Dynamics Analysis - Command Augmentation Design
GUI	Graphical User Interface
FDP	Flight Dynamics Principles , 3rd Edition

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